

The secret of all victory lies in the organization of the nonobvious.

Marcus Aurelius

After motivation and mental attitude, the most important limitation on your ability to solve problems is biological:

Working memory is 7 ± 2 "pieces of information".

You can't change this biological fact. All you can do is take advantage of your environment to get around it.

That means, you must put things into your environment to manipulate them.

Externalize: write things down, manipulate aspects of the problem (correct the representation of the problem).

A rubber ball has the property that, on any bounce, it returns to one-third of the height from which it just fell.

Suppose the ball is dropped from 108 ft.

How far has the ball travelled vertically the fourth time it hits the ground?

In this example, drawing the picture left your mind free to concentrate on problem solving.

Not drawing is probably hopeless, too much to keep track of.

To be effective, the drawing needs to be set up right – a diagram of some sort makes a big difference.

Remember these numbers:



Now, look away and multiply them in your head.

A rectangular board is sawed into two pieces by a straight cut across its width. The larger piece is twice the length of the smaller piece.

This smaller piece is cut again into two parts, one three times the length of the other. You now have three pieces of board. The smallest piece is a 7-inch square.

What was the original area of the surface of the board?

Problems along one dimension: distance, money, etc.

John has a pretty good salary. In fact if the salary of his older brother, Bob, were suddenly doubled, John would make only 100 dollars less than Bob. Bob's current salary is 50 dollars more than that of the youngest brother, Phil. John makes 600 dollars per week.

What is Phil's salary?

Draw a line and put the information onto the line.

Tom, Dick, Harry, and Al are married to May, Jane, Sue, and Bea, though not necessarily in that order.

Jane, who is Dick's sister, has five children.

Tom and his wife want to wait a few more years before starting a family.

Tom has never introduced his wife to Sue, who is carrying on an extramarital affair with Dick. (May is considering telling Dick's wife about it.)

Dick and Harry, by the way, are twin brothers.

Who is married to whom?

How can we organize this information?

Matrix works well in this case

Can work on one row/column (e.g., figure out who X is married to).

Can work one fact at a time.

In this case, we will get pretty far. But we'll be left with a 2 by 2 box for Harry/Al and Jane/Sue. How do we break it?

We need to relate two facts to infer that Dick, Harry, Jane are all siblings.

Example

Three boys, Joey, Jimmy, and Pete, have between them nine quarters and a total of \$2.55 in quarters and nickels.

Joey has three nickels, and Jimmy has the same number of quarters.

Jimmy has one coin more than Joey, who has four coins.

How many nickels each do Jimmy and Pete have?

The Southwest Virginia Chicken Fanciers Club holds an annual show and competition featuring some of their best exotic fowl. This year, there were three entrants from the Montgomery County area: Dwight, Robert, and Joe. Altogether, they entered 36 birds in the competition, including 17 Rhode Island Reds and 6 Scots Greys.

Norman entered a total of 9 birds, including 3 Rhode Island Reds and his only Leghorn.

Robert entered no Scots Greys.

Joe entered 7 Rhode Island Reds and 5 Leghorns.

How many chickens of each type did each of them enter in the show and competition?

An anthropologist and her husband attended a party with four other married couples.

Whenever two people shook hands, the woman recorded that each of the two people shook hands one time. In that way, for all of them (including herself and her husband), she obtained the total number of times that each person shook hands.

She noted that one didn't shake hands with one's own spouse. Then she observed: If she didn't count herself, the other nine people all shook hands a different number of times. That is, one person didn't shake any hands, one shook only once, up to one shaking hands of all eight of the others.

Q: How many times did her husband shake hands?

This one is difficult. Its tough to engage.

But there are things that can be figured out. You need to play with it awhile.

Hint: Can the anthropologist's husband be the one who shook hands 8 times?

Bigger hint: Draw out a table!

Let's arbitrarily let couple #1 be the anthropologist and her husband.

	C1H1	C1W1	C2H2	C2W2	C3H3	C3W3	C4H4	C4W4	C5H5	C5W5
C1H1	X	X								
C1W1	X	X								
C2H2			X	X						
C2W2			X	X						
C3H3					X	X				
C3W3					X	X				
C4H4							X	X		
C4W4							X	X		
C5H5									X	X
C5W5									X	X

No one shakes hands with his/her spouse or herself/himself., so mark those cells with X's.

i.e., we make an assumption that has no effect on the nature or existence of a solution

HS Problem: Case 8 Shakes

Suppose the anthropologist's husband shakes hands with 8 people:

	C1H1	C1W1	C2H2	C2W2	C3H3	C3W3	C4H4	C4W4	C5H5	C5W5
C1H1	X	X	👍	👍	👍	👍	👍	👍	👍	👍
C1W1	X	X								
C2H2	👍		X	X						
C2W2	👍		X	X						
C3H3	👍				X	X				
C3W3	👍				X	X				
C4H4	👍						X	X		
C4W4	👍						X	X		
C5H5	👍								X	X
C5W5	👍								X	X

Mark cells indicating each of those handshakes...

... but now we clearly see it's not possible for anyone (excluding the anthropologist) to shake hands with 0 people.

HS Problem: Case 7 Shakes

Suppose the anthropologist's husband shakes hands with 7 people; arbitrarily suppose he does not shake hands with C2H2:

	C1H1	C1W1	C2H2	C2W2	C3H3	C3W3	C4H4	C4W4	C5H5	C5W5
C1H1	X	X	X	👍	👍	👍	👍	👍	👍	👍
C1W1	X	X								
C2H2	X		X	X						
C2W2	👍		X	X						
C3H3	👍				X	X				
C3W3	👍				X	X				
C4H4	👍						X	X		
C4W4	👍						X	X		
C5H5	👍								X	X
C5W5	👍								X	X

HS Problem: Case 7 Shakes

Then C2H2 is the only eligible person who can shake hands with noone:

	C1H1	C1W1	C2H2	C2W2	C3H3	C3W3	C4H4	C4W4	C5H5	C5W5
C1H1	X	X	X	👍	👍	👍	👍	👍	👍	👍
C1W1	X	X	X							
C2H2	X	X	X	X	X	X	X	X	X	X
C2W2	👍		X	X						
C3H3	👍		X		X	X				
C3W3	👍		X		X	X				
C4H4	👍		X				X	X		
C4W4	👍		X				X	X		
C5H5	👍		X						X	X
C5W5	👍		X						X	X

HS Problem: Case 7 Shakes

But then, C2W2 is the only person who could have shaken hands with 8 people:

	C1H1	C1W1	C2H2	C2W2	C3H3	C3W3	C4H4	C4W4	C5H5	C5W5
C1H1	X	X	X	👍	👍	👍	👍	👍	👍	👍
C1W1	X	X	X	👍						
C2H2	X	X	X	X	X	X	X	X	X	X
C2W2	👍	👍	X	X	👍	👍	👍	👍	👍	👍
C3H3	👍		X	👍	X	X				
C3W3	👍		X	👍	X	X				
C4H4	👍		X	👍			X	X		
C4W4	👍		X	👍			X	X		
C5H5	👍		X	👍	But now, no eligible person could shake hands with only one person...				X	X
C5W5	👍		X	👍					X	X

Continue in like fashion, assuming the anthropologist 's husband shook hands with 6 people, then 5 people, until you find a case that does not lead to a contradiction.

Of course, you should also ask whether more than one case might avoid a contradiction...