Making an Argument

- The goal of communication is to achieve the desired affect on the target audience.
- Often we want to convince the audience of something
  - Answers on an exam
  - Making a proposal
  - Letter to the editor
- The goal is not to be right.
- The goal is to convince the audience that we are right.

Investigation and Argument

- How can we be convincing?
  - Need to be right (investigation/solution)
  - Need to present it right (argument)
- Part of good communication is to reduce cognitive load on the audience
- Good technical writing is essentially about making clear, logical arguments
- Following standard presentation forms can help.
  - Conventions in reasoning
  - Proof forms

Mathematical Proof

- "Mathematical" proofs often follow one of several standard forms
  - These forms have proved useful for structuring ideas
  - Following a conventional form reduces cognitive load on the reader

Deduction (Direct Proof)

- If P, then Q
- $P \rightarrow Q$
- Contrapositive: (not Q) $\rightarrow$ (not P)
- Sometimes can break this down:
  - Truth of the penultimate step $\rightarrow$ The conclusion

Reasoning Chains

- Many systems work by chaining a series of steps
  - Symbolic Logic
  - Geometry proofs
  - Calculus integrals

Contradiction

- Want to prove X
- Assume that X is false
- Show that this assumption leads to a logical contradiction
- Since the assumption must be false, X must be true

Contradiction Example

Prove that there is no largest integer

- Assume that there is a largest integer, B.
- Consider C = B + 1.
- C is an integer (the sum of two integers)
- C > B.
- Thus, B is not the largest integer, a contradiction.
- The only flaw in the reasoning was the assumption that there exists B, the largest integer.
- Therefore, there is no largest integer.

Contradiction Example

Prove that $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ is rational.
- $\sqrt{2} = a/b$ for a and b integers and b is as small as possible.
- Since $2b^2 = a^2$, a^2 is even (so a is even).
- So $a = 2t$, yielding $2b^2 = a^2 = 4t^2$.
- So $b^2 = 2t^2$, making b even.
- But then it is not possible for $\sqrt{2} = a/b$.

Mathematical Induction

- To prove by induction, must show two things:
  - **Base case**: We can get started
  - **Induction step**: Being true for n-1 implies that it is true also for n
- Often easy to prove base case
- Might or might not be easy to prove the induction step
  - Note that we are proving an implication:
  - $S(n-1) \rightarrow S(n)$

Induction Hypothesis

- The key to induction is the induction hypothesis.
- We assume S(n-1) is true.
- This gives us material to work with.
- It is also what confuses people most.

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Induction Example

Call S(n) the sum of the first n integers. Prove that S(n) = n(n+1)/2.

- **Base case**: S(1) = 1(1+1)/2 = 1, which is true.
- **Induction hypothesis**: S(n-1) = (n-1)n/2.
- **Induction step**: Use the induction hypothesis
  - S(n) = S(n-1) + n
  - S(n) = (n-1)n/2 + n = (n^2 – n + 2n)/2 = n(n+1)/2.
- Therefore, the theorem is proved by mathematical induction.

Induction Example

- 2-cent and 5-cent stamps can be used to form any value n ≥ 4.
- **Base case**: 2 + 2 = 4.
- **Induction hypothesis**: Assume true for any greater value n-1.
- Induction step:
  - Case i: A 5-cent stamp is replaced with 3 2-cent stamps.
  - Case ii: Two 2-cent stamps are replaced with a 5-cent stamp.
- Therefore, the theorem is proved by induction

Induction and Recursion

- Induction and Recursion are similar
- If you are comfortable with one, should quickly be able to grasp the other
- Both have a base case.
- Both use the assumption that subproblems are true/solvable
  - Recursion makes a recursive call
  - Induction uses the induction hypothesis
- A recursive function's primary work is converting solutions to subproblems into the full solution
  - This is the same as the induction step.

Other forms of Induction

- "Standard" induction: $S(n-1) \rightarrow S(n)$
- "Strong" induction: $S(1)$ to $S(n-1) \rightarrow S(n)$
  - Strong induction gives us a stronger induction hypothesis.
  - The induction hypothesis is free material to work with.

Guess and Test

- One approach to problem solving is to guess an answer and then test it.
- When finding closed forms for summations, can guess a solution and then test with induction.
- Induction can test a hypothesis, but doesn't help to generate a hypothesis.