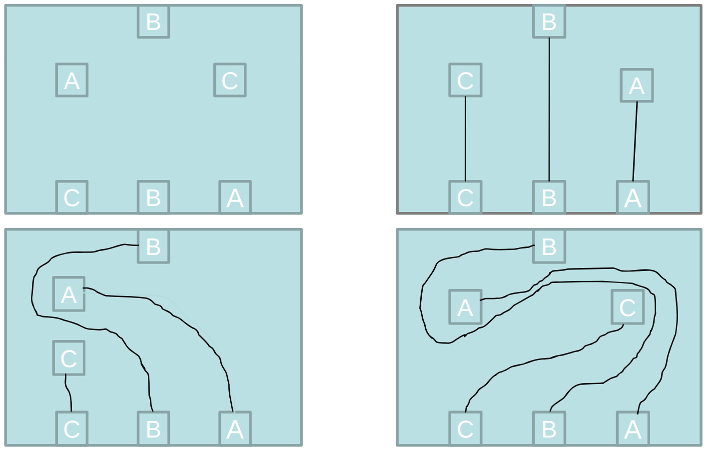
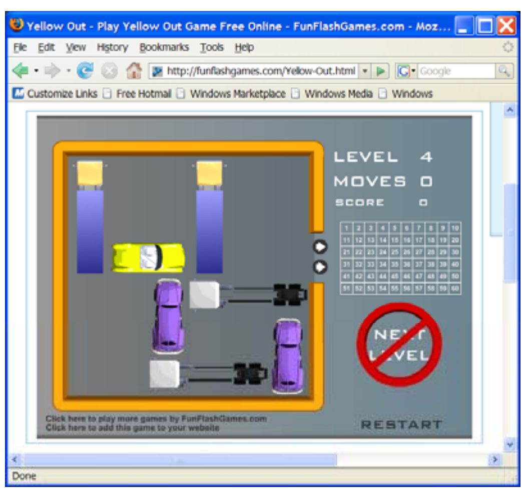
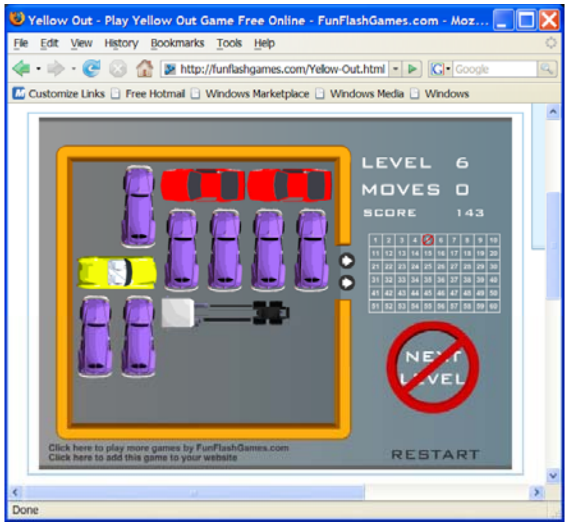
Heuristic: Wishful Thinking



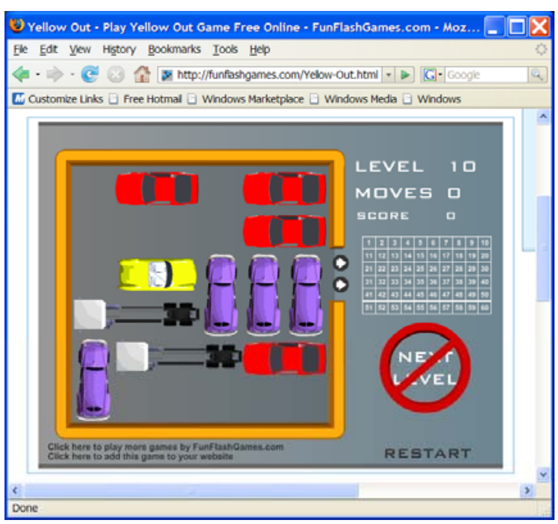
Heuristic: Penultimate Step



Another Puzzle



Another Puzzle



Monks Problem

A monk climbs a mountain. He starts from the bottom at 8 AM and reaches the top at noon. He spends the rest of the day there. The next day, he leaves at 8 AM and goes back to the bottom along the same path. Prove that there is a time between 8AM and noon on each day that he is in the same place, at the same time, on both days.

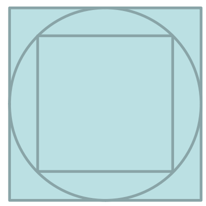
Stuck? Try drawing a picture.

Heuristic: Look for Symmetry

- If you find a symmetry, you might be able to exploit it
  - Symmetries give you “free” information, cut down on what to look at
  - Symmetries define an invariant
  - Symmetries indicate “special” points

Symmetry Problem

- What is the ratio of the areas of the two squares?



Symmetry Problem

Your cabin is two miles due north of a stream that runs east-west. Your grandmother's cabin is located 12 miles west and one mile north of your cabin. Every day, you go from your cabin to Grandma's, but first visit the stream (to get fresh water for Grandma). What is the length of the route with minimum distance?

Stuck? Draw a picture!

Symmetry Problem

What is the sum of the values 1 to 100?

Hint: Look for the symmetry!

The Pigeonhole Principle

If you have more pigeons than pigeonholes, when the pigeons fly into the holes at night, at least one hole has more than one pigeon.

Problem: Every point on the plane is colored either red or blue. Prove that no matter how the coloring is done, there must exist two points, exactly a mile apart, that are the same color.

Pigeonhole Problem

Given a unit square, show that if five points are placed anywhere inside or on this square, then two of them must be at most sqrt(2)/2 units apart.

Invariants

- An invariant is some aspect of a problem that does not change.
  - Similar to symmetry
  - Often a problem is easier to solve when you focus on the invariants

Motel problem: If G is amount paid by guests, P amount pocketed, and D amount held by the desk clerk, then G = P + D.

Invariant Problem

At first, a room is empty. Each minute, either one person enters or two people leave. After exactly 3^{1999} minutes, could the room contain $3^{1000} + 2$ people?

Invariant Problem

If 127 people play in a singles tennis tournament, prove that at the end of the tournament, the number of people who have played an odd number of games is even.