1. (5 points) Solve Exercise 3.4.6 (b) on page 89 of your textbook.

   **Solution:** A set of attributes is a superkey if it contains $A_1$ or $A_2$ or both. The set $\{A_1, A_2, \ldots, A_n\}$ has $2^n$ subsets. A set $S$ of attributes is not a superkey if it does not contain $A_1$ or $A_2$, i.e., $S$ is a subset of $\{A_3, A_4, \ldots, A_n\}$. Since $\{A_3, A_4, \ldots, A_n\}$ contain $n - 2$ elements, it has $2^{n-2}$ subsets. Therefore, the number of possible superkey is $2^n - 2^{n-2} = 3^n - 2^{n-2}$.

2. (15 points) Solve Exercise 3.5.2 (ii) on page 100 of your textbook. Note that you have to read Exercise 3.5.1 in order to solve this problem.

   **Solution:** Non-trivial FDs:
   
   $AB \rightarrow D$; $BC \rightarrow A$; $CD \rightarrow B$; $AD \rightarrow C$; $ABC \rightarrow D$; $BCD \rightarrow A$; $ACD \rightarrow B$; $ABD \rightarrow C$.

   Keys:
   
   $\{A, B\}$; $\{B, C\}$; $\{C, D\}$; $\{A, D\}$.

   Superkeys:
   
   $\{A, B, C\}$; $\{B, C, D\}$; $\{A, C, D\}$; $\{A, B, C, D\}$.

3. (15 points) Solve Exercise 3.5.10 (c) on page 102 of your textbook. You do not have to worry about the process used to create $S$. You only need to write down the completely non-trivial FDs that hold in $S$ and have single attributes on the right hand side.

   **Solution:** The closure of the FDs on $R$ also contains $BC \rightarrow A$, $AC \rightarrow B$. Both FDs only involve attribute of $S$. Therefore, FDs that hold in $S$ are: $AC \rightarrow B$ and $BC \rightarrow A$.

4. (20 points) Solve Exercise 3.6.1 (f) on page 117 of your textbook. You do not have to solve part (iv) of this question, i.e., you do not have to bring the relation into 3NF.

   **Solution:** The keys of $R$ are $\{A, B\}$; $\{A, C\}$; $\{A, D\}$.

   Therefore, the FDs that violate BCNF are: $C \rightarrow D$; $D \rightarrow B$; $D \rightarrow E$; $C \rightarrow B$; $C \rightarrow E$.

   Decomposition: Step 1) Using the FD $C \rightarrow E$: $\{C, E\}$; $\{A, B, C, D\}$
   Step 2) Using the FD $C \rightarrow D$: $\{C, E\}$; $\{C, D\}$; $\{A, B, C\}$
   Step 3) Using the FD $C \rightarrow B$: $\{C, E\}$; $\{C, D\}$; $\{C, B\}$; $\{A, C\}$ (Final result)

   Note that starting from another FD, eg, $D \rightarrow B$ yields a different decomposition.

   The FDs that violate 3NF are: $D \rightarrow E$ and $C \rightarrow E$.

5. (10 points) Solve Exercise 3.7.3 (d) on page 126 of your textbook.

   **Solution:** The key of $R$ is $\{A, B, C\}$.

   Therefore, MDs that violate 4NF are: $A \rightarrow B$, $AB \rightarrow C$, and $A \rightarrow C$.

   Decomposition: Step 1) Using the MD $AB \rightarrow C$: $\{A, B, C\}$; $\{A, B, D, E\}$
   Step 2) Using the MD $A \rightarrow B$: $\{A, B, C\}$; $\{A, B\}$; $\{A, D, E\}$
   Step 3) Using the MD $A \rightarrow B$: $\{A, C\}$; $\{A, B\}$; $\{A, D, E\}$ (Final result)

6. (20 points) A key is simple if it consists of only one attribute. Prove that if a relation $R$ is in 3NF and if every key in $R$ is simple, then $R$ is in BCNF. Your proof should be general, e.g., it should
not assume that $R$ has a fixed number, say two or three, attributes. The proof is simple; my proof has just three sentences.

**Solution:** By the definition of 3NF, since $R$ is in 3NF, every FD $\{A_1, A_2, \ldots, A_n\} \rightarrow B$ that holds in $R$:

(i) $\{A_1, A_2, \ldots, A_n\}$ is a superkey.

(ii) $B$ is an attribute in a key. Since every key in $R$ is simple, $B$ is a key, which implies that $\{A_1, A_2, \ldots, A_n\}^+$ is the set of all attributes of $R$, i.e., $\{A_1, A_2, \ldots, A_n\}$ is a superkey.

Therefore, $R$ is in BCNF.

7. (5 points) Every two-attribute relation is in 4NF. Is this statement true or false?

**Solution:** True, since any MD that holds in the relation is trivial.

8. (10 points) $R(A, B, C)$ is a relation in which the FD $B \rightarrow C$ and the MD $A \rightarrow B$ hold. This FD and MD impose a certain constraint on the tuples that can exist in $R$. State this constraint as succinctly as you can.

**Solution:** The constraint is $A \rightarrow C$. To prove this property, consider two tuples: $t_1 = (a, b_1, c_1)$ and $t_2 = (a, b_2, c_2)$. Since $A \rightarrow B$, $R$ must also contain the tuples $(a, b_1, c_2)$ and $(a, b_2, c_1)$. Since $B \rightarrow C$ holds in $R$, we must have that $c_1 = c_2$. Therefore, whenever two tuples have the same value of $A$, they must have the same value of $C$, i.e., $A \rightarrow C$. 