CS 4114
Homework Exercise 3

Given: February 13, 2001  Due: February 16, 2001

The point value of each problem is shown in [ ]. Your solutions may be created in \LaTeX. Your solutions must be created electronically and may be submitted in printed form (preferred) or electronically as a postscript or PDF file attached to email sent to cs4114@courses.cs.vt.edu.

For every solution submitted, a careful justification of your answer is required. Such a justification will typically consist of a proof or appeal to some theorem proved in class or in the book. It may also consist of explicitly carrying out the steps of an algorithm presented in class. The quality of your technical writing will be evaluated, so write carefully and completely. A solution, correct or incorrect, without a justification is worth no credit.

The assignment must be delivered to the instructor at McBryde 638 or received at the cs4114 account by 12:00 noon on February 16, 2001. If sent by email, the subject line should be “Solutions to Homework Exercise 3”. NO UNEXCUSED LATE HOMEWORKS WILL BE ACCEPTED. See syllabus for details.

If you submit a first version of your solutions and then decide to submit a revised version BEFORE THE DEADLINE, you may do so as follows. Before you send the revised version, send email to the GTA, Mr. Yang (xiyang@vt.edu), and send a copy to the course account (cs4114@courses.cs.vt.edu) with the subject line “Resubmission of Homework Exercise 3”. Explain briefly in the email what revisions you made. Immediately send the revision to cs4114@courses.cs.vt.edu with the subject line “Solutions to Homework Exercise 3: Resubmission”.

In this assignment, we will use the definition of \( n_\sigma(w) \), where \( \Sigma \) is any alphabet, \( \sigma \in \Sigma \), and \( w \in \Sigma^* \), to be the number of \( \sigma \)'s in \( w \). For example, \( n_c(acbcdca) = 3 \).

\[15\] 1. For each of the following languages over \( \Sigma_1 = \{a, b, c, d\} \), give a regular expression that represents the set.

a. \( L_a = \{ w \in \Sigma_1^* \mid n_c(w) > 2 \} \)

b. \( L_b = \{ w \in \Sigma_1^* \mid n_a(w) = 3 \text{ and } n_c(w) = 1 \} \)

c. \( L_c = \{ w \in \Sigma_1^* \mid \text{the string } bba \text{ occurs exactly once in } w \} \).
[15] 2. Using normal set notation (as in Problem 1), give a definition of the language $L_2 \subseteq \{0, 1\}^*$ represented by the regular expression

$$r_2 = (0^*(1(01^*0)^*)^*)^*0^*.$$  

**Hint:** First make a list of “small” strings represented by $r_2$. Think about what natural number each string represents in binary.

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[20] 3. Let $G_3$ be the following context-free grammar:

$$S \rightarrow AA$$

$$A \rightarrow AAA \mid a \mid bA \mid Ab.$$  

a. List all derivations in $G_3$ of length at most four that derive strings in $L(G_3)$.

b. Give four distinct derivations for the string $babab$.

c. For any $m, n, p > 0$, describe a derivation in $G_3$ of the string $b^mab^nab^p$.  

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