CS 4114
Homework Exercise 2

Given: February 3, 2001

The point value of each problem is shown in [ ]. Your solutions may be created in \LaTeX. Your solutions must be created electronically and may be submitted in printed form (preferred) or electronically as a postscript or PDF file attached to email sent to cs4114@courses.cs.vt.edu.

For every solution submitted, a careful justification of your answer is required. Such a justification will typically consist of a proof or appeal to some theorem proved in class or in the book. It may also consist of explicitly carrying out the steps of an algorithm presented in class. The quality of your technical writing will be evaluated, so write carefully and completely. A solution, correct or incorrect, without a justification is worth no credit.

The assignment must be delivered to the instructor at McBryde 638 or received at the cs4114 account by 12:00 noon on February 9, 2001. If sent by email, the subject line should be “Solutions to Homework Exercise 2”. NO UNEXCUSED LATE HOMEWORKS WILL BE ACCEPTED. See syllabus for details.

If you submit a first version of your solutions and then decide to submit a revised version BEFORE THE DEADLINE, you may do so as follows. Before you send the revised version, send email to the GTA, Mr. Yang (xiyang2@vt.edu), and send a copy to the course account (cs4114@courses.cs.vt.edu) with the subject line “Resubmission of Homework Exercise 2”. Explain briefly in the email what revisions you made. Immediately send the revision to cs4114@courses.cs.vt.edu with the subject line “Solutions to Homework Exercise 2: Resubmission”.

[20] 1. Give a careful proof by induction that \[n^4 - 4n^2\] is divisible by 3, for all \(n \geq 0\).

[15] 2. Give a recursive definition of the set of palindromes over the alphabet \(\Sigma_2 = \{a, b, c\}\).
[15] 3. Let $\Sigma_3 = \{a, b, c, d\}$. The following are languages over $\Sigma_3$.

$$L_1 = \{x \mid x = y y^R, \text{ for some } y \in \Sigma_3^*\}$$

$$L_2 = \{x \mid x = y y, \text{ for some } y \in \Sigma_3^*\}$$

$$L_3 = \{x \mid x x = y y y, \text{ for some } y \in \Sigma_3^*\}$$

For each of the three languages, give 2 examples of strings in the language; and 2 examples of strings not in the language.