Suppose that G is a directed graph with N vertices and E edges. The **out-degree** of a vertex v in G is the number of edges starting at v and ending at some other vertex u in G. Similarly, the **in-degree** of a vertex x is the number of edges starting at some other vertex u and ending at v. Given an adjacency-list representation of a directed graph:

1. Given an adjacency-list representation of the directed graph G, how many operations would it take to find the out-degree of every vertex in G?

   1) \( \Theta(N) \)  
   2) \( \Theta(E) \)  
   3) \( \Theta(N^2) \)  
   4) \( \Theta(E^2) \)  
   5) \( \Theta(N \times E) \)  
   6) \( \Theta(N^2 \times E^2) \)  
   7) \( \Theta(N + E) \)  
   8) \( \Theta(N^2 + E^2) \)  
   9) None of these

2. Given an adjacency-list representation of the directed graph G, how many operations would it take to find the in-degree of every vertex in G?

   1) \( \Theta(N) \)  
   2) \( \Theta(E) \)  
   3) \( \Theta(N^2) \)  
   4) \( \Theta(E^2) \)  
   5) \( \Theta(N \times E) \)  
   6) \( \Theta(N^2 \times E^2) \)  
   7) \( \Theta(N + E) \)  
   8) \( \Theta(N^2 + E^2) \)  
   9) None of these

For questions 3 through 6, consider the following directed graph H:

3. Suppose a depth-first traversal is performed on H, beginning at the vertex S. Adopting the convention that the choice of which neighbor to visit first is based upon alphabetical ordering of the neighbor labels, in what order will the vertices of H be visited?

   1) S A B C W X Y  
   2) S A B C Y X  
   3) S A B C X Y W  
   4) S A B W C  
   5) S A B W C X Y  
   6) S A B Y W C X  
   7) None of these

4. Suppose a breadth-first traversal is performed on H, beginning at the vertex S. Adopting the convention that the choice of which neighbor to visit first is based upon alphabetical ordering of the neighbor labels, in what order will the vertices of H be visited?

   1) S A B C W X Y  
   2) S A B C X Y  
   3) S A B C X Y W  
   4) S A B W C  
   5) S A B W C X Y  
   6) S A B Y W C X  
   7) None of these
5. Consider constructing a minimal-weight spanning tree for H, starting at the vertex S. What would the total weight of the spanning tree be?

1) 9  
2) 11  
3) 12  
4) There is no spanning tree for H.  
5) It depends on the algorithm used.  
6) None of these

6. Repeat question 5 for the graph obtained from H by reversing the edge (in H) from A to B.

1) 11  
2) 12  
3) 16  
4) There is no spanning tree for H.  
5) It depends on the algorithm used.  
6) None of these

7. Suppose L is a list of 10 elements and the probability p(k) that L[k] will be the target of a search operation is given by:

\[ p(k) = \begin{cases} 
\frac{1}{2^k} & 0 \leq k \leq 8 \\
\frac{1}{2^9} & k = 9 
\end{cases} \]

What is the average number of comparisons that will be performed when L is searched (to the closest integer)?

1) 0  
2) 1  
3) 2  
4) 3  
5) 4  
6) 5  
7) 6  
8) 7  
9) More than 7  
10) None of these

8. For the list described in question 8, which of the following statements are true? Assume searches are always for values that actually occur in the list. Interpret "about" to mean within ±1.

1) If many searches are performed, we would expect about 50% of them to require at most three comparisons.
2) If many searches are performed, we would expect over 90% of them to require at most three comparisons.
3) If many searches are performed, we would expect about 50% of them to require only one comparison.
4) If many searches are performed, we would expect about 5% of them to require more than seven comparisons.
5) 2 and 3 only
6) 3 and 4 only
7) 1 and 4 only
8) 1, 3 and 4 only
9) 2, 3 and 4 only
10) None of these