You will submit your solution to this assignment to the Curator System (as `ComplexityHW`). Your solution must be either a plain text file or a MS Word document.

1. Using the rules given in the course notes, perform an exact count complexity analysis of the body of the following function. (Your answer should be a function of the parameter N.)

```c
void Mystery(int M[N][N], const int N) {
    for (int R = 0; R < N; R++) {
        for (int C = 1; C < N; C++) {
            if (M[R][C-1] < M[R][C])
                M[R][C-1] = M[R][C];
            else {
                M[R][C] = M[R][C-1];
                M[R][C-1] = 0;
            }
        }
    }
}
```

Note: array index operations count 1 time unit each.

2. For each of the following inequalities, what is the smallest value of N such that the inequality holds for all n >= N?

(a) \(0.1n \geq 10 \log n\)

(b) \(\frac{1}{2}n^2 - n \leq 20n \log n\)

3. Divide the following functions into non-overlapping categories (\(\Theta\) equivalence classes), so that two functions, say \(f(n)\) and \(g(n)\), are in the same category if and only if \(f(n) \in \Theta(g(n))\). Arrange the categories from the lowest order of magnitude to the highest. A function may be in a category by itself, or there may be several functions in the same category.

- \(5000\)
- \(\log n\)
- \(n^2 \log n\)
- \(\log \log n^2\)
- \(\log n^2\)
- \((\log n)^5\)
- \(n + \log n\)
- \(n^2 - 100n\)
- \(n^{0.3}\)
- \((n^2 + 4)^{1/2}\)
- \(3^n\)
- \(n^3\)
- \(4n + n^{1/2}\)
- \(n^2\)
- \(2^n\)

4. Using any theorems from the course notes, give a formal proof that:

\[
(n - 5)(n + \log n + \sqrt{n}) \in \Theta(n^2)
\]
5. Decide if each of the following statements is true or false — no justification is necessary.

(a) \((3 \log n)^3 - 10\sqrt{n} + 2n\) is \(\Theta(n)\)

(b) \(\sqrt{n^2 - 10n + 100}\) is \(\Omega(n)\)

(c) \(2^n - n^3\) is \(\Omega(n^4)\)

6. Suppose that executing an algorithm on input of size \(N\) requires executing \(T(N) = N \log N + 8N\) instructions. How long would it take to execute this algorithm on hardware capable of carrying out \(2^{23}\) instructions per second if \(N = 2^{50}\)? (Give your answer in hours, minutes and seconds, to the nearest second.)