You will submit your solution to this assignment to the Curator System (as Homework2). Your solution must be either a plain text file or a MS Word document.

1. Using the rules given in the course notes, perform an exact count complexity analysis of the body of the following function. (Your answer should be a function of the parameter N.)

   ```cpp
   void Mystery(int M[N][N], const int N) {
       for (int R = 0; R < N; R++) {
           for (int C = 1; C < N; C++) {
               if (M[R][C-1] < M[R][C])
                   M[R][C-1] = M[R][C];
               else {
                   M[R][C] = M[R][C-1];
                   M[R][C-1] = 0;
               }
           }
       }
   }
   ```

   Note: any arithmetic operations for array index values DO count.

2. For each of the following inequalities, what is the smallest value of N such that the inequality holds for all n >= N?
   (a) \( n \log_{10} 1.0 \geq 0.1n \log n \)
   (b) \( \frac{1}{2} n^2 - n \geq 20n \log n \)

3. Divide the following functions into categories, so that two functions, say \( f(n) \) and \( g(n) \), are in the same category if and only if \( f(n) = \Theta(g(n)) \). Arrange the categories from the lowest order of magnitude to the highest. A function may be in a category by itself, or there may be several functions in the same category.

   - \( 5000 \)
   - \( \log n \)
   - \( n^2 \log n \)
   - \( \log \log n^2 \)
   - \( \log n^2 \)
   - \( (\log n)^5 \)
   - \( n + \log n \)
   - \( n^2 - 100n \)
   - \( n^{0.3} \)
   - \( n^{1/2} \)
   - \( n^3 \)
   - \( 4n + n^{1/2} \)
   - \( 2^n \)
   - \( n^2 \)

4. Using any theorems from Kruse/Ryba or the course notes, give a formal proof that:

   \[ (n - 5)(n + \log n + \sqrt{n}) \text{ is } \Theta(n^2) \]
5. Decide if each of the following statements is true or false — no justification is necessary.

(a) \((3 \log n)^3 - 10\sqrt{n} + 2n\) is \(\Theta(n)\)

(b) \(\sqrt{n^2} - 10n + 100\) is \(\Omega(n)\)

(c) \(2^n - n^3\) is \(\Omega(n^4)\)

6. Suppose that executing an algorithm on input of size \(N\) requires executing \(T(N) = 3N^2 + 7N\) instructions. How long, in seconds, would it take to execute this algorithm on hardware capable of carrying out \(10^7\) instructions per second if \(N = 10^9\)? (Give your answer to the nearest hundredth of a second.)