1. Define these functions of the natural numbers:

\[ g_1(n) = n^2 \]
\[ g_2(n) = 2^n \]
\[ g_3(n) = (\log n)^{\log n} \]

Which of these statements is true?

a. \( g_1 \in O(g_2) \) and \( g_2 \in O(g_3) \)

b. \( g_2 \in O(g_3) \) and \( g_3 \in O(g_1) \)

c. \( g_3 \in O(g_1) \) and \( g_2 \in O(g_1) \)

d. None of the above

\[ \text{Answer} \]

\[ \text{d. None of the above} \]

2. Define the function \( T(n) \) by the following recurrence:

\[ T(n) = 3T(n/4) + n^{2.5} \]

What is the easiest method to solve this recurrence asymptotically?

a. Master Theorem

b. Evaluate the summation

c. Recursion tree

d. None of the above

\[ \text{Answer} \]

\[ \text{a. Master Theorem} \]
3. Consider a hash table $T[0..m-1]$ with open addressing, where $m = 10$. Assume uniform hashing and an initially empty table. The expected number of probes to insert 2 keys into $T$ is
   a. 2
   b. $\frac{19}{20}$
   c. $\frac{21}{20}$
   d. None of the above

   **Answer**
   c. $\frac{21}{20}$

4. Let $X = x_1x_2\ldots x_m$ and $Y = y_1y_2\ldots y_n$ be two strings over the alphabet $\Sigma = \{A, C, G, T\}$. A longest common subsequence (LCS) of $X$ and $Y$ can be found by dynamic programming. To compute $c[i,j]$, where $1 \leq i \leq m$ and $1 \leq j \leq n$, how many table entries are examined in the worst case?
   a. 2
   b. 3
   c. $n$
   d. $n + m$
   e. None of the above

   **Answer**
   b. 3

5. You are driving alone from Virginia Beach to Los Angeles in a Porsche 911 Carrera on a fixed 2800 mile route. Assume that you can go 250 miles on one tank of gas and that you have a map showing every gas station on the route. What methodology for designing efficient algorithms would you choose to determine which gas stations to stop at and how much gas to get at each station?
   a. Divide and conquer
   b. Greedy
   c. Dynamic programming
   d. None of the above

   **Answer**
   b. Greedy

6. You are to design an algorithm to find a simple path in a graph from given vertex $s$ to given vertex $t$. That simple path is to contain as few edges as possible. Which of the following will you base your algorithm on?
   a. Depth-first search
b. Breadth-first search
c. Dijkstra’s algorithm
d. Kruskal’s algorithm
e. None of the above

**Answer**

b. Breadth-first search

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7. Let $G = (V, E)$ be a connected, undirected graph with edge weights $w : E \to \mathbb{Z}$. Suppose $G$ has a unique minimum spanning tree. What can you conclude about $G$?
   a. $G$ contains no cycles
   b. $G$ contains at most one cycle
   c. All edge weights are different
   d. None of the above

**Answer**
d. None of the above

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8. Let $G = (V, E)$ be a connected, undirected graph with edge weights $w : E \to \mathbb{Z}$. Which of the following statements are true about the Floyd-Warshall algorithm applied to $G$?
   a. Since $G$ is not directed, we cannot apply Floyd-Warshall.
   b. Since $G$ is undirected, Floyd-Warshall will be asymptotically faster than on directed graphs.
   c. Since $G$ is undirected, Floyd-Warshall will be unable to detect negative-weight cycles.
   d. None of the above

**Answer**
d. None of the above

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9. Which of the following decision problems is not NP-complete?
   a. Hamiltonian Path
   b. Shortest Path
c. 3-CNF Satisfiability
d. Set Cover
e. All of the above
f. None of the above

**Answer**

b. Shortest Path