Cuts

A cut is a set of cut events, one per node, each of which captures the state of the node on which it occurs.

\[ C = \{ c_1, c_2, c_3 \} \]

Consistent Cut

A cut \( C = \{ c_1, c_2, c_3, \ldots \} \) is consistent if for all sites there are no events \( e_i \) and \( e_j \) such that:

\[ (e_i \rightarrow e_j) \quad \text{and} \quad (e_j \rightarrow c_j) \quad \text{and} \quad (e_i \not\rightarrow c_i) \]

An inconsistent cut:
Ordering of Cut Events

The cut events in a consistent cut are not causally related. Thus, the cut is a set of concurrent events and a set of concurrent events is a cut.

Note, in this inconsistent cut, \( c_3 \rightarrow c_2 \).

Time of a Cut

Each cut event, \( c_i \), is assigned a vector timestamp, \( VT_{c_i} \). The cut set is assigned a vector timestamp \( VT_c \) where

\[
VT_c[i] = \max(VT_{c_1}[i], VT_{c_2}[i], \ldots, VT_{c_n}[i]).
\]

The cut is consistent iff:

\[
VT_c = (VT_{c_1}[1], VT_{c_2}[2], \ldots, VT_{c_n}[n])
\]

because this implies that no message sent after cut event \( c_i \) has been received before some other cut event \( c_j \).
Question:

In a distributed computation, when are all of the processes idle (i.e., when has the computation terminated)?

Huang’s Algorithm

The computation starts when the controlling agent sends the first message and terminates when all processes are idle.

The role of weights:
- the controlling agent initially has a weight of 1 and all others have a weight of zero,
- when a process sends a message, a portion of the sender’s weight is put in the message, reducing the sender’s weight,
- a receiver adds to its weight the weight of a received message,
- on becoming idle, a process sends its weight to the controlling agent,
- the sum of all weights is always 1.

The computation terminates when the weight of the controlling agent reaches 1 after the first message.
Rules

sending a message

active \( W \) \[\rightarrow\] active \( W_1 \) \[\rightarrow\] send \( W_2 \)

where \( W_1 + W_2 = W \)

receiving a message

\( W \) \[\rightarrow\] idle \( 0 \) \[\rightarrow\] active \( W \)

Rules

receiving a message

\( W_2 \) \[\rightarrow\] receive \[\rightarrow\] active \( W_1 \) \[\rightarrow\] active \( W \)

where \( W_1 + W_2 = W \)

becoming idle

active \( W \) \[\rightarrow\] idle \( 0 \) \[\rightarrow\] \( W \) \[\rightarrow\] controlling agent