Some NP-Complete Problems

SATISFIABILITY (SAT)
INSTANCE: A Boolean expression $E$ over variables $x_1, x_2, \ldots, x_n$ in conjunctive normal form.
QUESTION: Is there an assignment of truth values to $x_1, x_2, \ldots, x_n$ making $E$ true?

3-SAT
INSTANCE: A Boolean expression $E$ in conjunctive normal form such that each clause contains exactly 3 literals.
QUESTION: Is there a satisfying assignment for $E$?

3-COLORABILITY
INSTANCE: Graph $G = (V, E)$.
QUESTION: Is $G$ 3-colorable, that is, is there a function $f : V \rightarrow \{\text{red}, \text{blue}, \text{green}\}$ such that $f(u) \neq f(v)$ whenever $(u, v) \in E$?

3-DIMENSIONAL MATCHING (3DM)
INSTANCE: A set $M \subseteq W \times X \times Y$ where $W$, $X$, and $Y$ are disjoint sets having the same number $q$ of elements.
QUESTION: Does $M$ contain a matching, i.e., a subset $M' \subseteq M$ such that $|M'| = q$ and no two elements of $M'$ agree in any coordinate?

EXACT COVER BY 3-SETS (X3C)
INSTANCE: Finite set $X$ with $|X| = 3q$, $q$ an integer; collection $C$ of 3-element subset of $X$.
QUESTION: Does $C$ contain an exact cover for $X$, i.e., a subcollection $C' \subseteq C$ such that every element of $X$ occurs in exactly one member of $C'$?

PARTITION
INSTANCE: A finite set $A$, and a “size” $s(a) \geq 0$ defined for each $a \in A$.
QUESTION: Is there a subset $A' \subseteq A$ such that

$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)?$$
KNAPSACK
INSTANCE: Items 1, …, N with \( \text{size}(i) \geq 0 \) and \( \text{value}(i) \geq 0 \) defined for each item \( i \); integers \( M, K \geq 0 \).
QUESTION: Is there a subset \( S \subset \{1, \ldots, N\} \) such that
\[
\sum_{i \in S} \text{size}(i) \leq M
\]
and
\[
\sum_{i \in S} \text{value}(i) \geq K?
\]

CLIQUE
INSTANCE: Undirected graph \( G = (V, E) \), positive integer \( K \leq |V| \).
QUESTION: Does \( G \) have a clique of size \( K \) or more, i.e., a subset \( V' \subset V \) with \( |V'| \geq K \) such that every two vertices of \( V' \) are adjacent?

INDEPENDENT SET
INSTANCE: Undirected graph \( G = (V, E) \); positive integer \( K \leq |V| \).
QUESTION: Does \( G \) contain an independent set of size \( K \) or more, i.e., a subset \( V' \subset V \) such that \( |V'| \geq K \) and such that no two vertices of \( V' \) are adjacent?

VERTEX COVER (VC)
INSTANCE: Undirected graph \( G = (V, E) \); positive integer \( K \leq |V| \).
QUESTION: Is there a vertex cover of size \( K \) or less for \( G \), i.e., a subset \( V' \subset V \) such that \( |V'| \leq K \) and such that for each \( (u, v) \in E \), either \( u \in V' \) or \( v \in V' \)?

DOMINATING SET
INSTANCE: Undirected graph \( G = (V, E) \); positive integer \( K \leq |V| \).
QUESTION: Does \( G \) contain a dominating set of size \( K \) or less, i.e., a subset \( V' \subset V \) with \( |V'| \leq K \) such that for all \( u \in V - V' \) there is a \( v \in V' \) for which \( (u, v) \in E \)?

HAMILTONIAN CIRCUIT (HC)
INSTANCE: Undirected graph \( G = (V, E) \).
QUESTION: Does \( G \) contain a Hamiltonian circuit, i.e., a simple cycle of length \( |V| \)?

End of List