CS 4804 Midterm Exam

1. (10 points) Consider the following search space where the edges are labeled with the costs and the nodes are labeled with the evaluation of a heuristic function $h(n)$ applied to them. The initial state is A and the goal state is G. Write down the order of the states expanded by the A* search algorithm.

2. (30 points) For each of the constraint graphs below,
   - state whether it is arc-consistent;
   - if it is not arc-consistent, simplify the domains so that the graph becomes arc-consistent;
   - after maintaining arc-consistency, state how many solutions are possible.

   The nodes represent variables, the data inside the nodes denote the domains, and the edges are labeled with the constraints.

3. (20 points) The magic square puzzle is the problem of placing the numbers from 1 to 9 on a 3-by-3 grid such that each position is occupied by a unique number and the sum of the numbers along a row, column, or diagonal equals 15. One solution to this puzzle is given below:

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   8 1 6
   3 5 7
   4 9 2
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   Pose the magic square puzzle as a boolean satisfiability (SAT) problem in CNF form, i.e., the boolean expression must be a conjunction of disjunctions of propositional variables. For full credit, state what the variables are, how many there are, and identify the clauses they participate in. Give enough examples of the clauses to convince us that you understand what is going on.

4. (10 points) Assuming $Onions$, $Cutting$, and $Crying$ are propositional variables, state true or false for each of the following:
   - $Onions \wedge Cutting \models Onions \vee Crying$
   - $Onions \vee Cutting \models Onions \wedge Cutting$
   - $(Onions \Leftrightarrow Cutting) \wedge (Cutting \Leftrightarrow Crying) \models Onions \Leftrightarrow Cutting \Leftrightarrow Crying$
   - $Onions \Leftrightarrow Cutting \Leftrightarrow Crying \models (Onions \Leftrightarrow Cutting) \wedge (Cutting \Leftrightarrow Crying)$

5. (10 points) Given that P and Q are propositional variables, is $P \models (Q \Rightarrow P)$ true? Irrespective of your answer to the above, attempt to prove it by resolution-refutation. What do you learn?

6. (20 points) Given that 'horses are animals' prove that 'the head of a horse is the head of an animal.' Use only the following vocabulary:
   - horse(x): true when x is a horse.
   - headof(x, y): true when y is the head of x.
   - animal(x): true when x is an animal.

   For full credit, state the two given sentences in first order predicate logic (5 points), convert to clauses suitable for resolution-refutation (10 points), and prove a contradiction (5 points).