CS 4804 Homework 3
Solution Sketches

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1. **(20 points)** These are trivial; the easiest way to calculate them is with a truth table: (a) true; (b) false; (c) false; and (d) false.

2. **(20 points)** The given entailment does hold:

   $1 : P$

   $G : \neg(Q \Rightarrow P) = \neg(\neg Q \vee P) = Q \wedge \neg P$

   $G1 : Q$; $G2 : \neg P$

   resolve 1, $G2$ to *null*

3. **(20 points)** Assume the following predicate terminology:

   - $\text{took}(x, y, z)$: true when student x took class y in term z.
   - $\text{score}(x, y, z)$: true when student x got a score of z in calss y.
   - $\text{passed}(x, y)$: true when student x passed class y.

   Then the various statements can be asserted as:

   - Some students took French in Spring 2001.
     $\exists x : \text{took}(x, \text{French}, \text{Spring2001})$.
     You can also be pedantic about the plurality inherent in 'some students' and assert that there must be two different people x and y, satisfying the above predicate.
   - Every student who took French passes it.
     $\forall xy : \text{took}(x, \text{French}, y) \Rightarrow \text{passed}(x, \text{French})$
     Obviously there is some relationship between score and passed, which we do not state explicitly (as it is not given).
   - Only one student took Greek in Spring 2001.
     $\forall xy : (\text{took}(x, \text{Greek}, \text{Spring2001}) \wedge \text{took}(y, \text{Greek}, \text{Spring2001})) \Rightarrow (x = y)$
   - The best score in Greek is always higher than the best score in French.
     $\exists xm : \text{score}(m, \text{Greek}, x) \;\wedge\; (\forall yz : \text{score}(y, \text{Greek}, z) \Rightarrow (x \geq z))$
     $\qquad \wedge\; (\forall ab : \text{score}(a, \text{French}, b) \Rightarrow (x > b))$
     Here we are assuming that this statement is true even across terms.

4. **(20 points)** Assume the following predicate terminology:

   Policy(x) :- x is an insurance policy

   Person(x) :- x is a person

   Expensive(x) :- x is expensive

   Smart(x) :- x is smart

   Buys(x,y) :- x buys y

   Sells(x,y,z) :- x sells y to z

Insured(x) :- x is insured

e) $\forall xy$: Buys$(x, y) \wedge$ Person$(x) \wedge$ Policy$(y) \Rightarrow$ Smart(x)

f) $\forall xy$: Buys$(x, y) \wedge$ Person$(x) \wedge$ Policy$(y) \Rightarrow \neg$ Expensive(y)

g) $\exists x\ \forall yz$: Sells$(x, y, z) \wedge$ Person$(z) \wedge$ Policy$(y) \Rightarrow \neg$ Insured(z)

5. **(20 points)**

1 : $\forall x$: horse$(x) \Rightarrow$ animal(x)

In clausal form, this becomes

$\neg$ horse$(x) \vee$ animal(x)

G : $\forall xy$: (horse$(x) \wedge$ headof$(x, y)) \Rightarrow (\exists z$: animal$(z) \wedge$ headof$(z, y))$

In clausal form and negated, this gives:

$G1$: horse(H); $G2$: headof(H, E); $G3$: $\neg$ animal$(z) \vee \neg$ headof(z, E)

resolve 1, $G1$ to animal(H)

resolve animal(H), $G3$ to $\neg$ headof(H, E)

resolve $\neg$ headof(H, E), G2 to *null*