Extended Operators in SQL and Relational Algebra

T. M. Murali

September 16, 2009
Bags or Sets?

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- Real RDBMSs treat relations as bags of tuples.
  - A tuple can appear multiple times in a relation.
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- Real RDBMSs treat relations as bags of tuples.
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  - Performance is one of the main reasons;
So far, we have said that relational algebra and SQL operate on relations that are sets of tuples. Real RDBMSs treat relations as bags of tuples. A tuple can appear multiple times in a relation. Performance is one of the main reasons; duplicate elimination is expensive since it requires sorting. If we use bag semantics, we may have to redefine the meaning of each relation algebra operator.
Bag Semantics: Projection and Selection

- **Projection** ($\pi()$): process each tuple independently; a tuple may appear in the resulting relation multiple times.
- **Selection** ($\sigma()$): process each tuple independently; a tuple may appear in the resulting relation multiple times.
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$\pi_{A,B}(R)$
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Bag Semantics: Union, Intersection, and Difference

- $R \cup S$: if tuple $t$ appears $k$ times in $R$ and $l$ times in $S$, $t$ appears in $R \cup S$ $k + l$ times.
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\[
\begin{array}{c|c}
R & S \\
\hline
A & B \\
1 & 2 \\
1 & 2 \\
2 & 3 \\
2 & 3 \\
\end{array}
\]

\[
\begin{array}{c|c}
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R & S \\
\hline
A & B & A & B \\
\hline
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2 & 3 & 2 & 3 \\
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- **$R - S$:** If tuple $t$ appears $k$ times in $R$ and $l$ times in $S$, $t$ appears in $R - S$ $\max\{0, k - l\}$ times.
Bag Semantics: Products and Joins

- **Product (×):** If a tuple $r$ appears $k$ times in a relation $R$ and tuple $s$ appears $l$ times in a relation $S$, then the tuple $rs$ appears $kl$ times in $R \times S$.

- **Theta-join and Natural join (⋈):** Since both can be expressed as applying a selection followed by a projection to a product, use the semantics of selection, projection, and the product.
Extended Operators

- Powerful operators based on basic relational operators and bag semantics.
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- Duplicate elimination: turn a bag into a set by eliminating duplicate tuples.
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- Aggregation: used by the grouping operator and to manipulate/combine attributes.
- Extended projections: projection on steroids.
- Outerjoin: extension of joins that make sure every tuple is in the output.
Sorting

RA  $\tau_{A_1,A_2,...}(R)$.

SQL  `SELECT ... FROM ... WHERE ... ORDER BY A_1, A_2, ....`
Sorting

RA $\tau_{A_1, A_2, \ldots}(R)$.

SQL SELECT ... FROM ... WHERE ... ORDER BY $A_1, A_2, \ldots$.

- The result is a list of tuples in $R$ but with the tuples sorted by their values in attributes $A_1, A_2, \ldots$.
- In SQL, use DESC after an attribute to specify sorting in descending order; ASC is the default.
- If you use the result in another query, sorted order is lost.
Duplicate Elimination

\[ \delta(R) \text{ is the relation containing exactly one copy of each tuple in } R. \]

\[ \text{SELECT DISTINCT } \ldots \]
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SQL `SELECT DISTINCT ...`

- Duplicate elimination is *expensive*, since tuples must be sorted or partitioned.
- Set operations in SQL (UNION, INTERSECT, and EXCEPT) operate on *sets* of tuples, i.e., they first eliminate duplicates.
- To make these operators treat relations as bags, follow the operation with the keyword ALL.
Aggregation

- Operators that summarise or aggregate the values in a single attribute of a relation.
- Operators are the same in relational algebra and SQL.
- All operators treat a relation as a bag of tuples.
- \texttt{SUM}: computes the sum of a column with numerical values.
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- **SUM**: computes the sum of a column with numerical values.
- **AVG**: computes the average of a column with numerical values.
- **MIN** and **MAX**:
  - for a column with numerical values, computes the smallest or largest value, respectively.
  - for a column with string or character values, computes the lexicographically smallest or largest values, respectively.

- **COUNT**: computes the number of non-NULL tuples in a column.

In SQL, can use **COUNT(*)** to count the number of tuples in a relation.
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How do we answer the query “Count the number of classes and the total enrollment of the classes each department teaches”?
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- Can we answer the query using the operators discussed so far?
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How do we answer the query “Count the number of classes and the total enrollment of the classes each department teaches”? Can we answer the query using the operators discussed so far? We need to group the tuples of Teach by DeptName and then aggregate within each group. Use the grouping operator.
Example of Grouping in Relational Algebra

▶ How do we answer the query “Count the number of classes and total enrollment of the classes each department teaches”?
Example of Grouping in Relational Algebra

How do we answer the query “Count the number of classes and total enrollment of the classes each department teaches”?

1. Group Courses by DeptName.

\[ \gamma \text{L} (\text{Courses}), \text{where L is a list containing three elements:} \]

1. \text{DeptName}: the grouping attribute,
2. \text{COUNT(Number)} \rightarrow \text{NumCourses}: an aggregated attribute computing the count of the Number attribute in each group and naming the new attribute NumCourses,
3. \text{SUM(Enrollment)} \rightarrow \text{TotalEnrollment}: an aggregated attribute computing the total of the Enrollment attribute and naming the new attribute TotalEnrollment.\]
Example of Grouping in Relational Algebra

How do we answer the query “Count the number of classes and total enrollment of the classes each department teaches”?

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\[ \gamma_L(\text{Courses}) \]
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Example of Grouping Continued

- How do we answer the query “Count the number of classes and total enrollment of the classes each department teaches”?

The complete operator is $\gamma_{\text{DeptName}}, \text{COUNT(Number) } \rightarrow \text{NumCourses}, \text{SUM(Enrollment) } \rightarrow \text{TotalEnrollment}(\text{Courses})$. The schema of the new relation is (DeptName, NumCourses, TotalEnrollment). We can group by multiple attributes, create as many new attributes as necessary, and apply other operators to the result, since the grouping operator produces a relation.
Example of Grouping Continued

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How do we answer the query “Count the number of classes and total enrollment of the classes each department teaches”?

The complete operator is

\[ \gamma_{\text{DeptName}} \text{,} \text{COUNT(}\text{Number}\text{)} \rightarrow \text{NumCourses} , \text{SUM(}\text{Enrollment}\text{)} \rightarrow \text{TotalEnrollment}(\text{Courses}) \]

The schema of the new relation is

(DeptName, NumCourses, TotalEnrollment).

We can group by multiple attributes.

We can create as many new attributes as necessary.

We can apply other operators to the result, since the grouping operator produces a relation.
Grouping in SQL

- Syntax is much simpler than relational algebra.
- Use the `GROUP BY` clause after the `WHERE` clause or after the `FROM`, if there is no `WHERE` clause.
- List grouping attributes after `GROUP BY`.
- Use `SELECT` clause to aggregate attributes.

Example query:

```
SELECT DeptName, COUNT(Number) AS NumCourses, SUM(Enrollment) AS TotalEnrollment
FROM COURSES
GROUP BY DeptName;
```
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Grouping in SQL

SELECT DeptName, COUNT(Number) AS NumCourses, 
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FROM COURSES 
GROUP BY DeptName;

- Aggregated attributes are evaluated on a per-group basis.
- Only attributes mentioned in the GROUP BY clause may appear unaggregated in the SELECT clause, e.g., Number must have an aggregation operator applied to it.
- There need not be any aggregated attribute in the SELECT clause.
- Read Chapter 6.4.6 of the textbook about affect of NULL values on grouping and aggregation.
Restricting Grouping in SQL

How do we answer the query “Count the number of classes each department teaches, restricted to departments that have total enrollment at least 500 in their classes (the classes taught by that department)”?

```sql
SELECT DeptName, COUNT(Number) AS NumCourses
FROM COURSES
GROUP BY DeptName
HAVING SUM(Enrollment) >= 500;
```
How do we answer the query “Count the number of classes each department teaches, restricted to departments that have total enrollment at least 500 in their classes (the classes taught by that department)”?

Need to introduce the HAVING clause

SELECT DeptName, COUNT(Number) AS NumCourses
FROM COURSES
GROUP BY DeptName
HAVING SUM(Enrollment) >= 500;
Rules for **HAVING** Clauses

- An aggregation in a **HAVING** clause applies only to the group being tested.
- If an attribute appears unaggregated in a **HAVING** clause, it must appear in the **GROUP BY** line.
**Complete SELECT Statement**

```
SELECT Attribute list
FROM Relation list
WHERE Condition or Subquery
GROUP BY Attribute list
HAVING Condition or Subquery
ORDER BY Attribute list;
```
<<\begin{quote}
\textbf{Complete \texttt{SELECT} Statement}
\end{quote}\>

\begin{verbatim}
SELECT Attribute list 
FROM Relation list 
WHERE Condition or Subquery 
GROUP BY Attribute list 
HAVING Condition or Subquery 
ORDER BY Attribute list;
\end{verbatim}

\begin{itemize}
  \item \texttt{WHERE} is evaluated \textit{before} \texttt{GROUP BY} and \texttt{HAVING}.
\end{itemize}
Joins in Relational Algebra and SQL

- Cross product:
  \[
  RA \quad R \times S \\
  SQL \quad R \text{ CROSS JOIN } S;
  \]

- Theta join:
  \[
  RA \quad R \bowtie_C S \\
  SQL \quad R \text{ JOIN } S \text{ ON } C;
  \]

- Natural join:
  \[
  RA \quad R \bowtie S \\
  SQL \quad R \text{ NATURAL JOIN } S;
  \]
Outer Joins

- A *dangling tuple* is one that fails to pair with any tuple in the other relation in a join operation.
- Outer joins allow dangling tuples to be included in the result of join operations, by padding them with NULL values.
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\[
\text{RA } R \bowtie S
\]

\[
\text{SQL } R \text{ NATURAL FULL OUTER JOIN } S;
\]

- Contains all tuples in \( R \bowtie S \).
- Includes every tuple in \( R \) that is not joined with a tuple in \( S \), after padding a special null symbol \( \bot \) (NULL in case of SQL).
- Same condition applied to \( S \).
A dangling tuple is one that fails to pair with any tuple in the other relation in a join operation. Outer joins allow dangling tuples to be included in the result of join operations, by padding them with NULL values.

\[
\begin{align*}
\text{RA} & \quad R \bowtie S \\
\text{SQL} & \quad R \text{ NATURAL FULL OUTER JOIN } S; \\
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Left outer join:

\[
\begin{align*}
\text{RA} & \quad R \bowtie_L S \\
\text{SQL} & \quad R \text{ NATURAL LEFT OUTER JOIN } S; \\
\end{align*}
\]

- Like \( R \bowtie S \) but ignores dangling tuples in \( S \).
Outer Joins

- A *dangling tuple* is one that fails to pair with any tuple in the other relation in a join operation.
- Outer joins allow dangling tuples to be included in the result of join operations, by padding them with NULL values.

\[
\text{RA} \quad R \bowtie S \\
\text{SQL} \quad R \text{ NATURAL FULL OUTER JOIN } S;
\]

- Contains all tuples in \( R \bowtie S \).
- Includes every tuple in \( R \) that is not joined with a tuple in \( S \), after padding a special null symbol \( \perp \) (NULL in case of SQL).
- Same condition applied to \( S \).

- Left outer join:

\[
\text{RA} \quad R \bowtie_L S \\
\text{SQL} \quad R \text{ NATURAL LEFT OUTER JOIN } S;
\]

- Like \( R \bowtie S \) but ignores dangling tuples in \( S \).

- Right outer join is analogous to left outer join.
- All outerjoin operators have theta-join analogues.