1. Exponentially Weighted Moving Average Change Detection (EWMACD)

EWMACD, proposed in Brooks et al. (2014), is a kernel regression approach modeling the time series as a linear combination of trigonometric polynomials. The model is trained over data collected in the initial two (or more) years. When the observations in the subsequent years deviate from the values forecast by the model for a 'substantial' length of time (persistence), a change is declared (recovery or disturbance). The training period as well as the persistence are parameters of the algorithm. A positive flag is raised in instances of sustained growth while sustained losses are indicated by negative flags. EWMACD is able to capture seasonal changes, but is also very sensitive to several algorithm parameters.

Algorithm EWMACD.

for row $r := 1$ **step** 1 **until** R

for column $c := 1$ **step** 1 **until** C **do**

Step 1: Write the time series data in the column $(D_{rc})_{\cdot b}$ as

$$(D_{rc})_{\cdot b} = \begin{pmatrix} u \\ v \end{pmatrix},$$

where the M-dimensional vector u is deemed training data and the $(S - M)$-dimensional vector v as the test data. Let

$$X = \begin{bmatrix} 1 & \sin t_1 & \cos t_1 & \cdots & \sin Kt_1 & \cos Kt_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \sin t_M & \cos t_M & \cdots & \sin Kt_M & \cos Kt_M \end{bmatrix}$$

be the Gram matrix for the time points $t_1, \ldots, t_M$, using K harmonics, where $M > 2K + 1$. The least squares fit to the training data u is then written as

$$u(t) = \alpha_0 + \sum_{i=1}^{K} (\alpha_{2i-1} \sin it + \alpha_{2i} \cos it)$$

with coefficients

$$\alpha = \left(X^t X\right)^{-1} X^t u$$

and residual

$$E(\alpha) = u - X\alpha.$$

Remark 1. In practice α is computed via a QR factorization of X, not by computing $(X^t X)^{-1}$ explicitly.

Next let $I = \{i \mid |E(\alpha)_i| < \gamma_1\}$, where γ_1 is a user defined threshold and $|I| > 2K + 1$. Calculate the coefficients for an improved fit to the underlying signal as

$$\alpha^* = \left((X_{I\cdot})^t X_{I\cdot}\right)^{-1} (X_{I\cdot})^t u_I.$$

With the refined coefficients α^*, calculate the residuals for

(i) the complete time series $(D_{rc})_{\cdot b}$ as

$$E^*(\alpha^*) = (D_{rc})_{\cdot b} - \bar{X}\alpha^*,$$

where $\bar{X}_{s\cdot} = (1, \sin t_s, \cos t_s, \ldots, \sin Kt_s, \cos Kt_s)$, for $s = 1, \ldots, S$.

(ii) the outlier-free time series as $\left(E^*(\alpha^*)\right)_{\bar{I}}$, where $\bar{I} = \{s \mid |E^*(\alpha^*)_s| < \gamma_2\}$, γ_2 is a user defined threshold, and

(iii) the outlier-free training set $\hat{I} = \bar{I} \cap \{1, \ldots, M\}$ as

$$\left(E^*(\alpha^*)\right)_{\hat{I}} = u_{\hat{I}} - X_{\hat{I}\cdot}\alpha^*,$$

where $|\hat{I}| > 2K + 1$.

Remark 2. In the present implementation,

$$\gamma_2 = \begin{cases} 1.5\eta, & i \in [1, M], \\ 20\eta, & i \in (M, S], \end{cases}$$

where η is the standard deviation of the first M elements of the residual vector $E^*(\alpha^*)$.

Step 2: Define the control limit vector τ by

$$\tau_i = \mu + \sigma L \sqrt{\frac{\lambda}{2-\lambda}\big(1-(1-\lambda)^{2i}\big)},$$

for $i = 1, 2, \ldots, |\bar{I}|$, where $\mu = 0$ is used here, σ is the standard deviation of the outlier-free training data errors $\big(E^*(\alpha^*)\big)_{\hat{I}}$, L is the multiple of this standard deviation σ, and $\lambda \in (0, 1]$ is the weight given to the most recent residual in the exponentially weighted moving average (EWMA) defined next. L is typically set to 3 or slightly smaller depending on the value of λ.

Step 3: Let $\bar{I} = \{j_1, j_2, \ldots, j_{|\bar{I}|}\}$, $j_1 < j_2 < \cdots < j_{|\bar{I}|}$. Define the vector z by

$$\begin{aligned} z_1 &= \big(E^*(\alpha^*)\big)_{j_1}, \\ z_i &= (1-\lambda)z_{i-1} + \lambda\big(E^*(\alpha^*)\big)_{j_i}, \quad i = 2, \ldots, |\bar{I}|. \end{aligned}$$

This is the exponentially weighted moving average (EWMA) of the residual $\big(E^*(\alpha^*)\big)_{\bar{I}}$.

Step 4: Define the flag history S-vector f by

$$f_s = \begin{cases} \operatorname{sgn}(z_i)\big\lfloor |z_i/\tau_i| \big\rfloor, & s = j_i \in \bar{I}, \\ 0, & \text{otherwise.} \end{cases}$$

If there is a run of $+1$ or -1 in the values $\operatorname{sgn}(\Delta f_s) = \operatorname{sgn}(f_{s+1} - f_s)$ of length ϖ, called the 'persistence', signal a change at the index s beginning the (nonzero) run.

Remark 3. Missing data is automatically handled by not assuming that the time points t_i are equally spaced. Alternatively, missing data for time point t_k can be handled by including t_k in the sequence $(t_1, t_2, \ldots, t_S)$, but excluding t_k from the training sequence $(t_1, t_2, \ldots, t_M)$ and k from the sets I, $\bar{I}$, and $\hat{I}$, which is equivalent to treating $(D_{rc})_{kb}$ as an outlier and to setting the flag $f_k = 0$.

end

end

The complexity of this approach is $\mathcal{O}\big((2K+1)^2 S\big)$. EWMACD outcomes are expected to be zeros when the time series trajectory matches the trajectory in the training period. Nonzeros are expected to appear when the trajectory deviates from the training period for a substantial length of time (determined by control chart parameters and the persistence). The first timepoint when the outcome trajectory deviates from the prior constant (or zero) value is the estimated date of disturbance, the subsequent nonzero sequence of values provides recovery (or, loss) period information. A sharp loss (e.g., harvest) is expected to manifest as a sudden drop in the outcome as well. A gentler loss (e.g., mountain pine beetle) is expected to appear in the form of the outcome trajectory gradually drifting away from zero (with an overall negative slope). Similarly, recovery will appear in the form of the trajectory following an overall positive slope, revealing recovery period information. In Brooks et al.(2017), Edyn, an improved version of EWMACD that retrains data after a disturbance is sensed, is proposed (this development not implemented here). In any case, since the training data in the postshock period is unstable, the training period of Edyn after the first break is expected to be unstable. For all the experiments presented in this work, the first two years of data is used for training EWMACD. The rest of the parameter values for EWMACD are as follows: $K = 2$, $L = 0.5$, $\lambda = 0.3$, $\varpi = 7$.