CS/MATH 4414 Homework #3

This homework is a Mathematica challenge, involving function definition and advanced graphics. The functions to be plotted are B-splines, so first some background on B-splines is required.

Definition. Let $t = (t_i)$ be a nondecreasing sequence (finite, infinite, or biinfinite). The *ith B-spline of order k for the knot sequence t* is denoted by $B_{i,k,t}$ and is defined by

$$B_{i,k,t}(x) = (t_{i+k} - t_i)\,(\tau - x)_+^{k-1}\big[t_i, \ldots, t_{i+k}\big], \qquad \text{all } x \in E.$$

(The divided difference is applied to $(\tau - x)_+^{k-1}$ considered as a function of τ.) If k and t are understood, write B_i instead of $B_{i,k,t}$.

Properties of B-splines:

(i) $B_i(x) = 0$ for $x \notin \big[t_i, t_{i+k}\big]$.

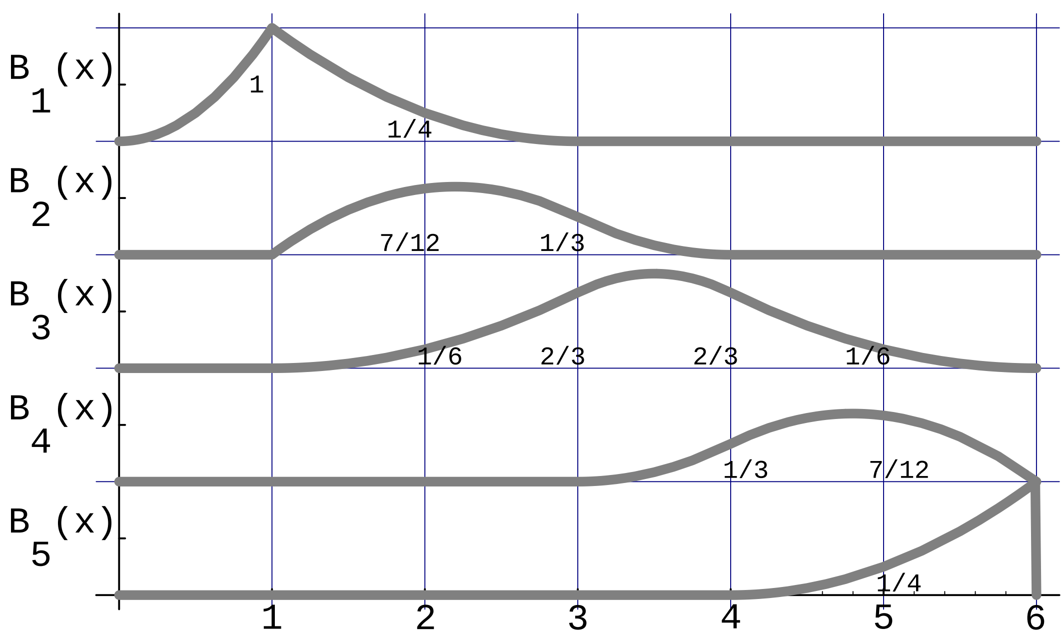
(ii) $\displaystyle\sum_i B_i(x) = \sum_{i=r+1-k}^{s-1} B_i(x) = 1$ for all $t_r < x < t_s$.

(iii) $B_i(x) > 0$ for $t_i < x < t_{i+k}$.

(iv) $\displaystyle B_{i,k}(x) = \frac{x - t_i}{t_{i+k-1} - t_i} B_{i,k-1}(x) + \frac{t_{i+k} - x}{t_{i+k} - t_{i+1}} B_{i+1,k-1}(x),$

where $B_{j,1}(x) = \begin{cases} 1, & t_j \le x < t_{j+1}, \\ 0, & \text{otherwise.} \end{cases}$ This property can be taken as the definition of $B_{i,k}$, but then the other properties of B-splines are not so easy to prove.

Your first task is to write and verify (by checking against, e.g., linear or cubic B-splines in textbooks) a Mathematica B-spline function `bspl[i,k,t,x]`. Then, for order $k = 3$ and knot sequence $t = (0,\ 1,\ 1,\ 3,\ 4,\ 6,\ 6,\ 6)$, produce the graph below using Mathematica.



Parabolic B-splines $B_{i,3}(x)$ for the knot sequence $t = (0, 1, 1, 3, 4, 6, 6, 6)$.

Each detail (labelling, grids, line shading, etc.) is to be reproduced as faithfully as you can. Turn in your Mathematica source for the function `bspl[i,k,t,x]`, for the plot, and the plot output. Relevant Mathematica graphics primitives are `Plot`, `Evaluate`, `Table`, `PlotStyle`, `Thickness`, `GrayLevel`, `DisplayFunction`, `GridLines`, `Ticks`, `SequenceForm`, `Subscript`, `StyleForm`, `FontFamily`, `FontSize`, `Text`, `Axes`.