Definition. Let \( t = (t_i) \) be a nondecreasing sequence (finite, infinite, or biinfinite). The \( ith \) B-spline of order \( k \) for the knot sequence \( t \) is denoted by \( B_{i,k,t} \) and is defined by

\[
B_{i,k,t}(x) = (t_{i+k} - t_i)(\tau - x)^{k-1}_+[t_i, \ldots, t_{i+k}], \quad \text{all } x \in E.
\]

(The divided difference is applied to \((\tau - x)^{k-1}_+\) considered as a function of \( \tau \).) If \( k \) and \( t \) are understood, write \( B_i \) instead of \( B_{i,k,t} \).

Properties of B-splines:

(i) \( B_i(x) = 0 \) for \( x \notin [t_i, t_{i+k}] \).

(ii) \( \sum_i B_i(x) = \sum_{i=r+1-k}^{s-1} B_i(x) = 1 \) for all \( t_r < x < t_s \).

(iii) \( B_{i,k}(x) = 0 \) for \( t_i < x < t_{i+k} \).

(iv) \( B_{i,k}(x) = \frac{x - t_i}{t_{i+k} - t_i} B_{i,k-1}(x) + \frac{t_{i+k} - x}{t_{i+k} - t_{i+1}} B_{i+1,k-1}(x) \),

where \( B_{j,1}(x) = \begin{cases} 1, & t_j \leq x < t_{j+1}, \\ 0, & \text{otherwise}. \end{cases} \) This property can be taken as the definition of \( B_{i,k} \), but then the other properties of B-splines are not so easy to prove.

Your first task is to write and verify (by checking against, e.g., linear or cubic B-splines in textbooks) a Mathematica B-spline function \( \text{bspl}[i,k,t,x] \). Then, for order \( k = 3 \) and knot sequence \( t = (0, 1, 1, 3, 4, 6, 6, 6) \), produce the graph below using Mathematica.

Parabolic B-splines \( B_{i,3}(x) \) for the knot sequence \( t = (0, 1, 1, 3, 4, 6, 6, 6) \).
Each detail (labelling, grids, line shading, etc.) is to be reproduced as faithfully as you can. Turn in your Mathematica source for the function \texttt{bspl[i,k,t,x]}, for the plot, and the plot output. Relevant Mathematica graphics primitives are \texttt{Plot}, \texttt{Evaluate}, \texttt{Table}, \texttt{PlotStyle}, \texttt{Thickness}, \texttt{GrayLevel}, \texttt{DisplayFunction}, \texttt{GridLines}, \texttt{Ticks}, \texttt{SequenceForm}, \texttt{Subscript}, \texttt{StyleForm}, \texttt{FontFamily}, \texttt{FontSize}, \texttt{TestListPlot}, \texttt{Axes}. 