Overview of 3D rendering

**Modeling:**
- *Define object in local coordinates*
- *Place object in world coordinates (modeling transformation)*

**Viewing:**
- Define camera parameters
- Find object location in camera coordinates (viewing transformation)

**Projection:** *project object to the viewplane*

**Clipping:** *clip object to the view volume*

- *Viewport transformation*
- *Rasterization: rasterize object*

* Topic we've already discussed

Simple teapot demo
3D rendering pipeline

Vertices as input

Series of operations/transformations to obtain 2D vertices in screen coordinates

These can then be rasterized
3D rendering pipeline

We've already discussed:
- Viewport transformation
- 3D modeling transformations

We'll talk about remaining topics in reverse order:
- 3D clipping (simple extension of 2D clipping)
- 3D projection
- 3D viewing
Clipping: 3D Cohen-Sutherland

*Use 6-bit outcodes*

*When needed, clip line segment against planes*
Viewing and Projection

**Camera Analogy:**

1. Set up your tripod and point the camera at the scene (viewing transformation).
2. Arrange the scene to be photographed into the desired composition (modeling transformation).
3. Choose a camera lens or adjust the zoom (projection transformation).
4. Determine how large you want the final photograph to be - for example, you might want it enlarged (viewport transformation).
Projection transformations
Introduction to Projection Transformations

Mapping: \( f : \mathbb{R}^n \rightarrow \mathbb{R}^m \)

Projection: \( n > m \)

Planar Projection: Projection on a plane.
- \( \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) or
- \( \mathbb{R}^4 \rightarrow \mathbb{R}^3 \) homogeneous coordinates.
Introduction to Projection Transformations

World Coordinates

Object in World

3D:2D mapping

2D:2D mapping

Viewport

Device Coordinates
## Classical Viewing

**Viewing requires three basic elements**
- One or more objects
- A viewer with a projection surface
- Projectors that go from the object(s) to the projection surface

**Classical views are based on the relationship among these elements**
- The viewer picks up the object and orients it how she would like to see it

**Each object is assumed to constructed from flat principal faces**
- Buildings, polyhedra, manufactured objects
Planar Geometric Projections

**Standard projections project onto a plane**

*Projectors are lines that either*
- converge at a center of projection
- are parallel

*Such projections preserve lines*
- but not necessarily angles

**Non-planar projections are needed for applications such as map construction**
Classical Projections

- Front elevation
- Elevation oblique
- Plan oblique
- Isometric
- One-point perspective
- Three-point perspective
Perspective vs Parallel

- Computer graphics treats all projections the same and implements them with a single pipeline.
- Classical viewing developed different techniques for drawing each type of projection.
- Fundamental distinction is between parallel and perspective viewing even though mathematically parallel viewing is the limit of perspective viewing.
Perspective Projection

Object

Projector

Projection plane

COP
Taxonomy of Planar Geometric Projections

Planar Geometric Projections

Parallel
- Multi-view Orthographic
  - Isometric
  - Dimetric
  - Trimetric
- Axonometric
- Oblique
- Perspective
  - 1 point
  - 2 point
  - 3 point
Examples
Orthographic Projection

Projectors are orthogonal to projection surface
Multiview Orthographic Projection

*Projection plane parallel to principal face*

*Usually form front, top, side views*

In CAD and architecture, we often display three multiviews plus isometric.
### Advantages and Disadvantages

<table>
<thead>
<tr>
<th>Preserves both distances and angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Shapes preserved</td>
</tr>
<tr>
<td>• Can be used for measurements</td>
</tr>
<tr>
<td>– Building plans</td>
</tr>
<tr>
<td>– Manuals</td>
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</tbody>
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**Cannot see what object really looks like because many surfaces hidden from view**

• Often we add the isometric
Axonometric Projections

Allow projection plane to move relative to object

classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric
Types of Axonometric Projections

Dimetric

Trimetric

Isometric
Advantages and Disadvantages

- **Lines are scaled (foreshortened) but can find scaling factors**
- **Lines preserved but angles are not**
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- **Can see three principal faces of a box-like object**
- **Some optical illusions possible**
  - Parallel lines appear to diverge
- **Does not look real because far objects are scaled the same as near objects**
- **Used in CAD applications**
Oblique Projection

Arbitrary relationship between projectors and projection plane
Advantages and Disadvantages

Can pick the angles to emphasize a particular face

- Architecture: plan oblique, elevation oblique

Angles in faces parallel to projection plane are preserved while we can still see “around” side

In physical world, cannot create with simple camera; possible with bellows camera or special lens (architectural)
Perspective Projection

Projectors converge at center of projection
Vanishing Points

- Parallel lines (not parallel to the projection plane) on the object converge at a single point in the projection (the vanishing point).
- Drawing simple perspectives by hand uses these vanishing point(s).
Three-Point Perspective

No principal face parallel to projection plane
Three vanishing points for cube
Two-Point Perspective

On principal direction parallel to projection plane
Two vanishing points for cube
One-Point Perspective

*One principal face parallel to projection plane*

*One vanishing point for cube*
Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (diminution)
  - Looks realistic
- Equal distances along a line are not projected into equal distances (non-uniform foreshortening)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)
A vertex located at $P$ in eye coordinates is projected to a certain point $(x^*, y^*)$ on the near plane, and is then mapped to the viewport on the display.
Perspective Projection Properties

- **Straight lines project to straight lines.** Consider the line between A and B. A projects to A' and B projects to B'.

- **In between:** consider the plane formed by A, B, and the origin. Since any two planes intersect in a straight line, this plane intersects the near plane in a straight line. Thus line segment AB projects to line segment A'B'.
Example: horizontal grid in perspective
Mathematics of a basic perspective projection

Similar triangles

\[
\frac{x'}{d} = \frac{x}{z}
\]

\[
\frac{y'}{d} = \frac{y}{z}
\]

\[
x' = \frac{x}{(z/d)}
\]

\[
y' = \frac{y}{(z/d)}
\]

\[
z' = d
\]
One-Point Projection in matrix form

Center of Projection at the origin with viewplane parallel to the x-y plane a distance \( d \) from the origin.

\[ \begin{align*}
    x_{\text{projected}} &= \frac{x}{z/d} \\
    y_{\text{projected}} &= \frac{y}{z/d}
\end{align*} \]

\[
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 1/d & 0
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    z \\
    z/d
\end{bmatrix}
= \begin{bmatrix}
    x/(z/d) \\
    y/(z/d) \\
    z \\
    1
\end{bmatrix}
\]

**Points plotted are \( x/w, y/w \) where \( w = z/d \)**