CS 4124
Converting an FSM to a Circuit

September 22, 2005

Finite State Machine Example

Figure 1 contains the state diagram of the example deterministic finite state machine M from September 20.

An Encoding for the FSM in Binary

To obtain a logic circuit for M, we need to express its inputs, outputs, and states as binary strings. Since $\Sigma = \Psi = \mathcal{B} = \{0, 1\}$, we first need to encode the four states of M in $\mathcal{B}^2$. We encode the state q_i as a pair $(b, c) \in \mathcal{B}^2$, where bc is the binary encoding of i. Figure 2 illustrates this encoding.

We also need to express δ and λ as binary functions. Since we need $\delta : Q \times \Sigma \to Q$ and since we encode Q with $\mathcal{B}^2$, we need δ to be a function $\delta : \mathcal{B}^2 \times \mathcal{B} \to \mathcal{B}^2$, and we write $\delta(b, c, \delta) = (b', c')$. Figure 3 contains δ expressed this way.

Since we need $\lambda : Q \to \Psi$ and since we encode Q with $\mathcal{B}^2$, we need λ to be a function $\delta : \mathcal{B}^2 \to \mathcal{B}$, and we write $\lambda(b, c) = \sigma$. Figure 4 contains λ expressed this way.

A Circuit for the FSM

To build a circuit for the FSM, we need, as a basic unit, a circuit to compute the ith computational step of the FSM, which amounts to computing

$$(b_i, c_i) \quad = \quad \delta(b_{i-1}, c_{i-1}, x_i)$$

and

$$y_i \quad = \quad \lambda(b_i, c_i).$$

To obtain an SOPE for b_i, we start with DNF for the b' column of Figure 3 and simplify:

$$\begin{aligned} b_i &= (\overline{b_{i-1}} \wedge \overline{c_{i-1}} \wedge \overline{x_i}) \vee (\overline{b_{i-1}} \wedge c_{i-1} \wedge \overline{x_i}) \vee (\overline{b_{i-1}} \wedge c_{i-1} \wedge x_i) \vee (b_{i-1} \wedge \overline{c_{i-1}} \wedge x_i) \\ &= (\overline{b_{i-1}} \wedge \overline{x_i}) \vee (\overline{b_{i-1}} \wedge c_{i-1}) \vee (b_{i-1} \wedge \overline{c_{i-1}} \wedge x_i). \end{aligned}$$

To obtain an SOPE for c_i, we start with DNF for the c' column of Figure 3 and simplify:

$$\begin{aligned} c_i &= (\overline{b_{i-1}} \wedge \overline{c_{i-1}} \wedge \overline{x_i}) \vee (\overline{b_{i-1}} \wedge \overline{c_{i-1}} \wedge x_i) \vee (b_{i-1} \wedge \overline{c_{i-1}} \wedge \overline{x_i}) \vee (b_{i-1} \wedge \overline{c_{i-1}} \wedge x_i) \\ &= \overline{c_i}. \end{aligned}$$

To obtain an SOPE for y_i, we use the DNF for the $\lambda(b, c)$ column of Figure 4:

$$c_i \quad = \quad (\overline{b_i} \wedge \overline{c_i}) \vee (b_i \wedge c_i).$$

Finally, we obtain the basic circuit in Figure 5.

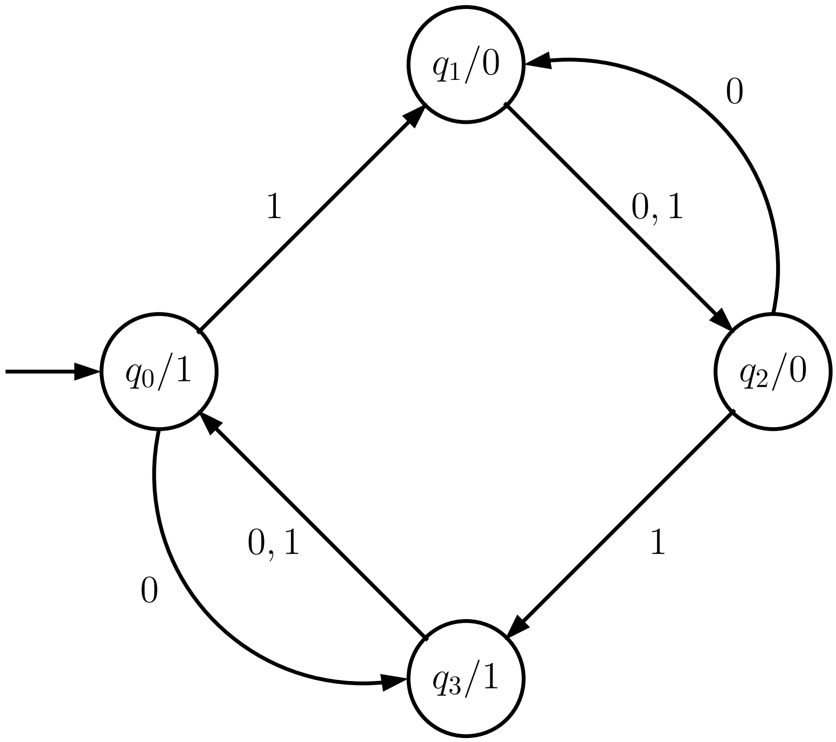

Figure 1: State diagram for M.

q	b	c
q_0	0	0
q_1	0	1
q_2	1	0
q_3	1	1

Figure 2: Encoding for each state q_i of M as a pair $(b, c) \in \mathcal{B}^2$.

b	c	σ	b'	c'
0	0	0	1	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

Figure 3: Transition table for M, expressed as $\delta(b, c, \sigma) = (b', c')$ for M.

b	c	$\lambda(b, c)$
0	0	1
0	1	0
1	0	0
1	1	1

Figure 4: Output table for M expressed as $\lambda(b, c)$.

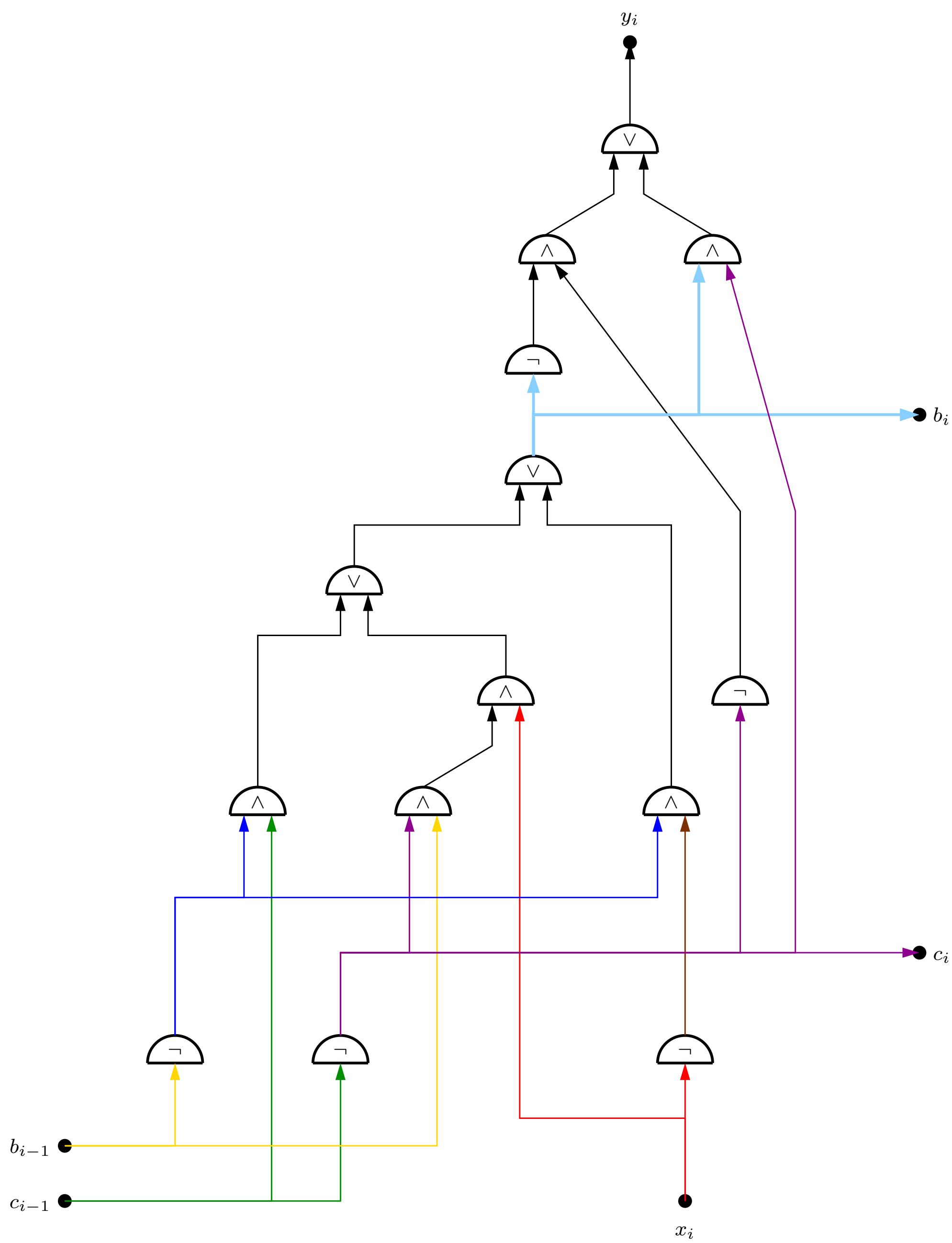

Figure 5: A circuit for M.