(15) 1. What important conclusion follows from the following two facts? (1) The unbounded memory RAM can simulate any Turing machine. (2) There is a Turing machine which can simulate the computation of any unbounded memory RAM program.

(25) 2. Consider a language \( L \in \mathbf{P} \), the TM \( M \) recognizing whether or not a string \( w \in L \) in \( p(|w|) \) steps, and some given string \( z \). The computation of \( M \) on input \( z \) takes \( T = p(|z|) \) steps, and can be simulated by a circuit \( C_{M,T} \). A description of this circuit \( C_{M,T} \) can be generated from \( M \) and \( z \) by a program \( \mathcal{P} \). Explain exactly why the program \( \mathcal{P} \) requires \( \mathcal{O}(\log n) \) space to execute, where \( n = |z| \).

(20) 3. A string \( w \) that reads the same left to right as right to left is called a palindrome. The language of palindromes, \( L = \{ w \mid w \in \Gamma^*, \ w = w^R \} \), over the alphabet \( \Gamma \) is not regular, and therefore is not accepted by a FSM. However, \( L \) is accepted by a nondeterministic pushdown automaton (PDA). Explain why \( L \) can not be accepted by a deterministic PDA.
(20) 4. In the proof that the language SATISFIABILITY (strings representing POSEs of Boolean functions that are 1 for some arguments) is \( \textbf{NP} \)-complete, a string describing a circuit (for CIRCUIT SAT) must be reduced to a string of clauses (a POSE for SATISFIABILITY). For instance, the instruction \((i \text{ OR } j \text{ OR } k)\) for the OR gate taking inputs \(g_j\) and \(g_k\) and producing output \(g_i = g_j \lor g_k\) must be reduced to an equivalent POSE in \(g_i, g_j, g_k\). Give this POSE (the first clause is provided as a hint).

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(g_i \lor \overline{g}_j) \]

(10) 5. State the generalization of the pumping lemma for regular languages required to prove that \(L = \{a^i b^j \mid i > j\}\) is not regular.

(10) 6. Which of the following languages are regular (answer yes/no)?

\[\begin{align*}
\ _a & \text{ a) } \{w \in \Sigma^* \mid \text{length of } w \text{ is odd}\} \\
\ _b & \text{ b) } \{w \in \{a, b\}^* \mid w \text{ has } ab \text{ and } ba \text{ as substrings}\} \\
\ _c & \text{ c) } \{w \in \{a, b\}^* \mid w \text{ has twice as many } a\text{'s as } b\text{'s}\} \\
\ _d & \text{ d) } \{a^n b a^n \mid n \geq 0\} \\
\ _e & \text{ e) } \{w \mid w \text{ is decimal notation for an integer that is a multiple of } 5\}
\end{align*}\]