CS 4124 FIRST EXAM **NAME** ____________________

Put your answers on these pages; use the back sides if necessary.

(20) 1. Show which of the order relationships— $f(n) = \mathcal{O}(g(n))$, $f(n) = \Omega(g(n))$, $f(n) = \Theta(g(n))$ —applies to the functions $f(n) = 3^n/n$, $g(n) = 2^n$.

(20) 2. In dual-rail logic each variable x is represented by the pair $(x, \bar{x})$. Using the standard AND, OR, and NOT gates, show how to build the corresponding dual-rail logic gates DRL-AND, DRL-OR, and DRL-NOT.

(20) 3. Prove that the RSE of a Boolean function $f : \mathcal{B}^n \to \mathcal{B}$ is unique.

(20) 4. The counting function $f^{(n)}_{\text{count}} : \mathcal{B}^n \rightarrow \mathcal{B}^{\lceil \log_2(n+1) \rceil}$ ($f(x) \equiv$ the number of 1s in $x_1, \ldots, x_n$) was implemented using a ripple adder on circuits for $f^{((n-1)/2)}_{\text{count}}$ (where $n = 2^{\ell+1} - 1$). Show how it is also possible to evaluate $f^{(n)}_{\text{count}}$ using a parallel prefix operation:

$$f^{(n)}_{\text{count}}(x_1, \ldots, x_n) = x_1 \odot x_2 \odot \cdots \odot x_n.$$

Precisely, what associative operation $\odot$ (and corresponding circuit) could be used?

(20) 5. Give the simplest SOPE representation of the Boolean function f defined by the truth table

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$f(x_1, x_2, x_3) =$ ____ .