CS4104 Spring 2007 Homework Assignment 7 Due at 11:00pm on Tuesday, March 13 50 Points

1. [10 points] Why is the following argument wrong?

"We have shown that n-1 comparisons are necessary to find the maximum of n values. By symmetry, n-1 comparisons are necessary to find the minimum. Therefore, 2n-2 comparisons are necessary to find the minimum and maximum.

2. [25 points] Give closed-form solutions for the following full-history recurrences. Be sure to explain why your closed-form solution is correct.

(a)

$$f(n) = \begin{cases} 1 & n = 1\\ \sum_{i=1}^{n-1} f(i) + 1 & n > 1 \end{cases}$$

(b)

$$f(n) = \begin{cases} 1 & n = 1\\ \sum_{i=1}^{n-1} f(i) + n & n > 1 \end{cases}$$

(c)

$$f(n) = \begin{cases} 1 & n = 1\\ \sum_{i=1}^{n-1} i f(i) + 1 & n > 1 \end{cases}$$

3. [15 points] Given a list of n elements, an element of the list is a *majority* if it appears more than n/2 times. Design an algorithm that is linear in the number of element-element comparisons in the worst case that will find a majority if one exists, and report that there is no majority if no such element exists.