

# CS4104 Spring 2007 Homework Assignment 4

Due at 11:00pm on Tuesday, February 13

50 Points

[Revised 2/13/2006]

1. [25 points]

- (a) Use induction to show that  $n^2 - n$  is always even.
- (b) Find a one-line proof of the same result.
- (c) Show that  $n^3 - n$  is always divisible by three.
- (d) Is  $n^5 - n$  always divisible by 5?

2. [25 points] Consider a variation on linear search in an unordered array that first checks the middle position of the array. If the element in this position is not equal to the search key, then linear search is called recursively on the lower half (not including the middle position) and the upper half (not including the middle position).

- (a) Write a recursive algorithm to implement this version of linear search. Be careful get the computation for the middle element correct, and to pass the bounds correctly on the recursive calls!
- (b) Show the recurrence relation for the algorithm. Be sure to show the EXACT values for the size of the subproblems in the recurrence (they are NOT quite  $\frac{n}{2}$ ).
- (c) Using your choice of either **substitute and guess** or **guess and test**, find a candidate closed form solution for the recurrence you gave in part (b).
- (d) Use induction to prove that your candidate from part (c) is correct.