

## CS4104 Spring 2007 Homework Assignment 12

Due at 11:00pm on Tuesday, May 1

50 Points

1. [15 points] A *Hamiltonian cycle* in graph  $\mathbf{G}$  is a cycle that visits every vertex in the graph exactly once before returning to the start vertex. The problem HAMILTONIAN CYCLE asks whether graph  $\mathbf{G}$  does in fact contain a Hamiltonian cycle. Assuming that HAMILTONIAN CYCLE is  $\mathcal{NP}$ -complete, prove that the decision-problem form of TRAVELING SALESMAN is  $\mathcal{NP}$ -complete.

2. [20 points] Define the problem PARTITION as follows:

PARTITION

**Input:** A collection of integers.

**Output:** YES if the collection can be split into two such that the sum of the integers in each partition sums to the same amount. NO otherwise.

Define the problem BIN PACKING as follows.

BIN PACKING:

**Input:** Numbers  $x_1, x_2, \dots, x_n$  between 0 and 1, and an unlimited supply of bins of size 1 (no bin can hold numbers whose sum exceeds 1).

**Output:** An assignment of numbers to bins that requires the fewest possible bins.

1. Assuming that PARTITION is  $\mathcal{NP}$ -complete, prove that BIN PACKING is  $\mathcal{NP}$ -complete.
2. Assuming that PARTITION is  $\mathcal{NP}$ -complete, prove that KNAPSACK is  $\mathcal{NP}$ -complete.

3. [15 points] We define the problem INDEPENDENT SET as follows.

INDEPENDENT SET

**Input:** A graph  $\mathbf{G}$  and an integer  $k$ .

**Output:** YES if there is a subset  $\mathbf{S}$  of the vertices in  $\mathbf{G}$  of size  $k$  or greater such that no edge connects any two vertices in  $\mathbf{S}$ , and NO otherwise.

Assuming that CLIQUE is  $\mathcal{NP}$ -complete, prove that INDEPENDENT SET is  $\mathcal{NP}$ -complete.