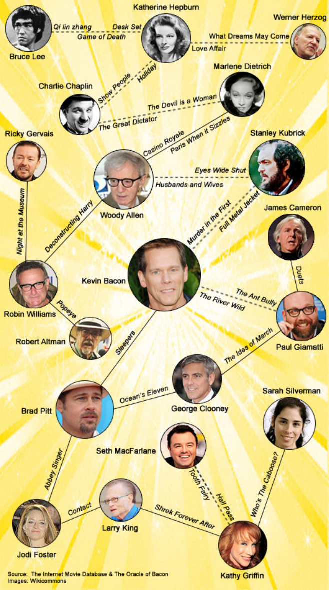
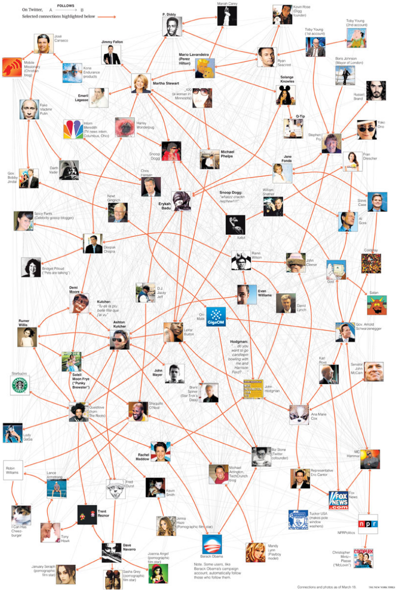
Graphs

T. M. Murali

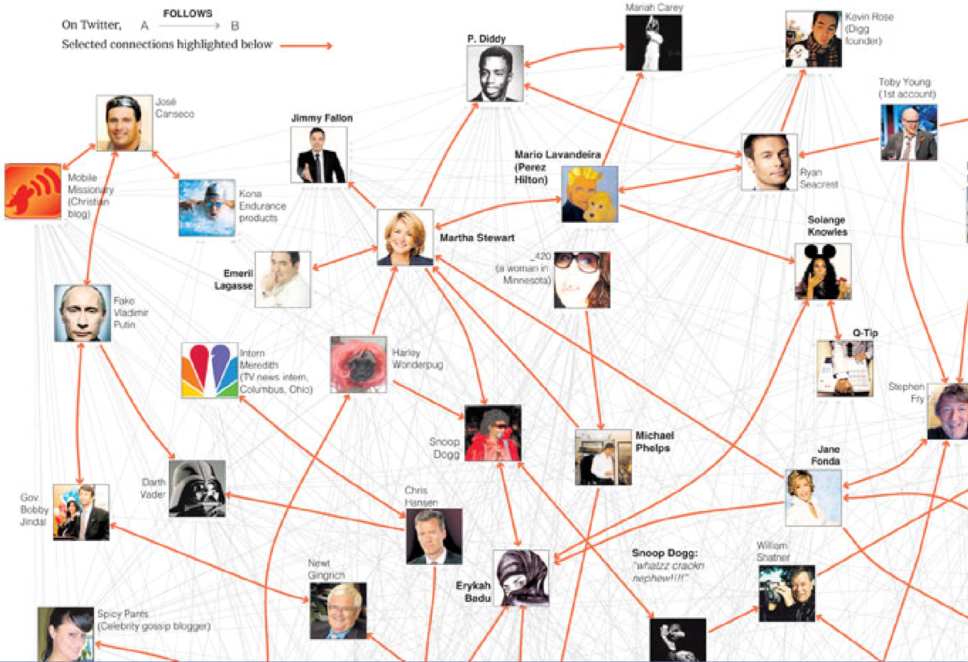
February 1, 3, 6, 8, 2017



The Oracle of Bacon



FOLLOWS
On Twitter, A → B
Selected connections highlighted below
P. Diddy
Mariah Carey
Kevin Rose (Digg founder)
Toby Young (1st account)
Toby Young (2nd account)
José Canseco
Jimmy Fallon
Mario Lavandeira (Perez Hilton)
Ryan Seacrest
Mobile Missionary (Christian blog)
Kona Endurance products
Martha Stewart
Boris Johnson (Mayor of London)
Solange Knowles
Emeril Lagasse
_420 (a woman in Minnesota)
Russell Brand
Fake Vladimir Putin
Q-Tip
Yoko Ono
Intern Meredith (TV news intern, Columbus, Ohio)
Harley Wonderpug
Stephen Fry
Snoop Dogg
Michael Phelps
Fran Drescher
Jane Fonda
Gov. Bobby Jindal
Darth Vader
Chris Hansen
Snoop Dogg: "whatzz crackn nephew!!!!"
William Shatner
Steve Case
Erykah Badu
Newt Gingrich
Al Gore
Spicy Pants (Celebrity gossip blogger)
Xzibit
Deepak Chopra
Rainn Wilson
John Cleese
Coldplay
God
Bridget Pilloud ("Pets are talking")
D.J. Jazzy Jeff
Evan Williams
Satan
Demi Moore
Kutcher: "Tu es la plu belle fille que j'ai vu"
David Lynch
Om Malik
GigaOM
Gov. Arnold Schwarzenegger
Rumer Willis
Ashton Kutcher
LeVar Burton
Hodgman: "... do you want to go candlepin bowling with me and Harrison Ford? ..."
Karl Rove
Senator John McCain
John Mayer
Soleil Moon Frye ("Punky Brewster")
Starbucks
Brent Spiner (Star Trek's Data)
John Hodgman
Questlove (from The Roots)
Shaquille O'Neal
Ana Marie Cox
Lady GaGa
Rachel Maddow
Biz Stone (Twitter cofounder)
MC Hammer
Robin Williams
Lance Armstrong
Fred Durst
Michael Arrington, TechCrunch blog
Representative Eric Cantor
Kevin Smith
Fox News
I Can Has Cheez-burger
Trent Reznor
Tucker USA (makes pole window washers)
Tony Hawk
Jenna Haze (Pornographic film star)
NPRPolitics
Dave Navarro
Barack Obama
Mandy Lynn (Playboy model)
Christopher Mintz-Plasse ("McLovin")
January Seraph (pornographic film star)
Joanna Angel (pornographic film star)
Sasha Grey (pornographic film star)
Note. Some users, like Barack Obama's campaign account, automatically follow those who follow them.
Connections and photos as of March 18.
THE NEW YORK TIMES



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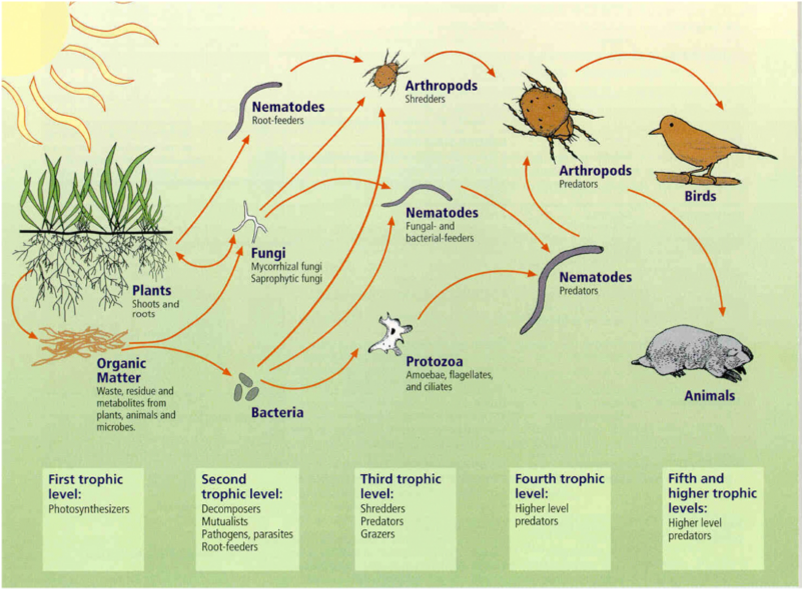
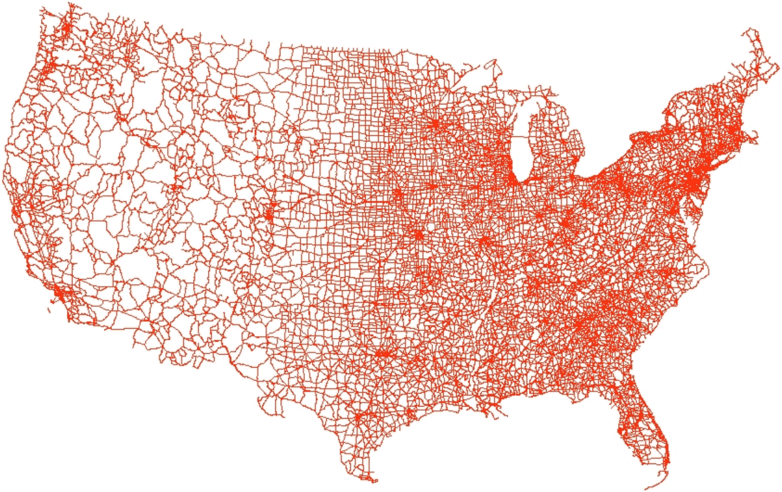
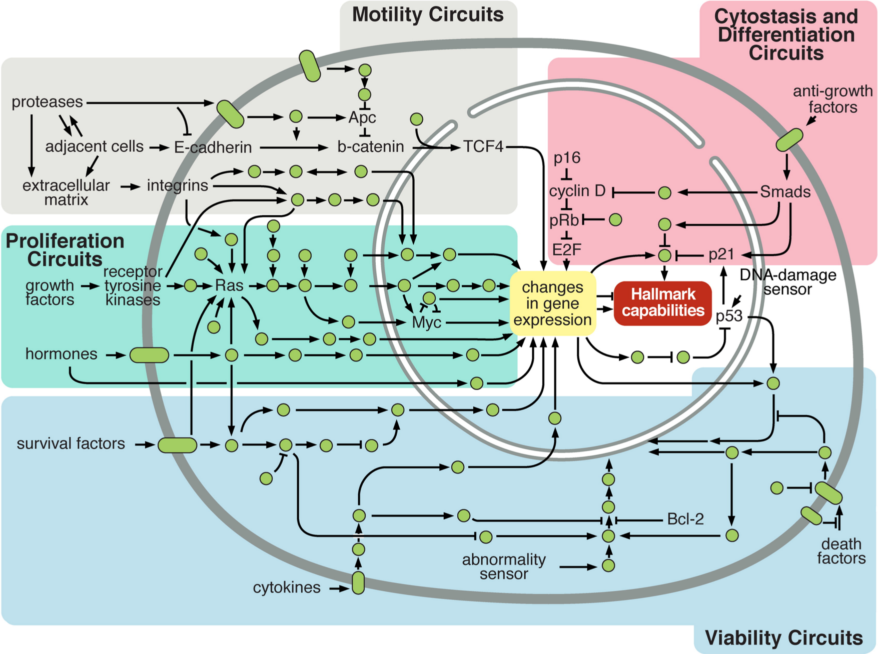

Nematodes
Root-feeders
Arthropods
Shredders
Arthropods
Predators
Birds
Plants
Shoots and roots
Fungi
Mycorrhizal fungi
Saprophytic fungi
Nematodes
Fungal- and bacterial-feeders
Nematodes
Predators
Organic Matter
Waste, residue and metabolites from plants, animals and microbes.
Bacteria
Protozoa
Amoebae, flagellates, and ciliates
Animals
First trophic level:
Photosynthesizers
Second trophic level:
Decomposers
Mutualists
Pathogens, parasites
Root-feeders
Third trophic level:
Shredders
Predators
Grazers
Fourth trophic level:
Higher level predators
Fifth and higher trophic levels:
Higher level predators




Motility Circuits
Cytostasis and Differentiation Circuits
proteases
adjacent cells
E-cadherin
extracellular matrix
integrins
Apc
b-catenin
TCF4
anti-growth factors
p16
cyclin D
pRb
E2F
Smads
p21
Proliferation Circuits
growth factors
receptor tyrosine kinases
Ras
Myc
changes in gene expression
Hallmark capabilities
DNA-damage sensor
p53
hormones
survival factors
Bcl-2
abnormality sensor
cytokines
death factors
Viability Circuits

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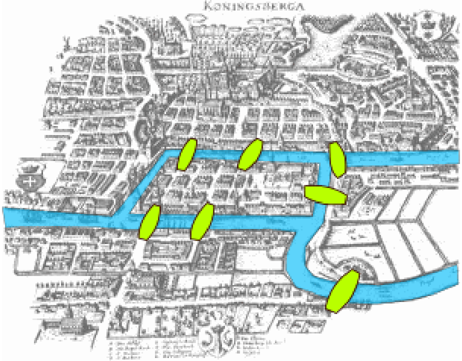

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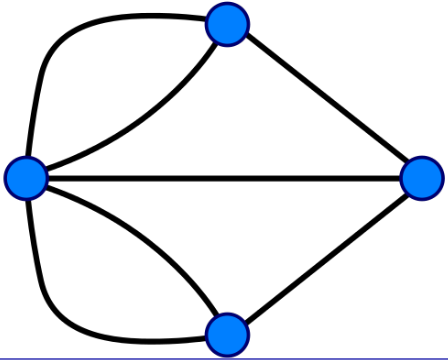
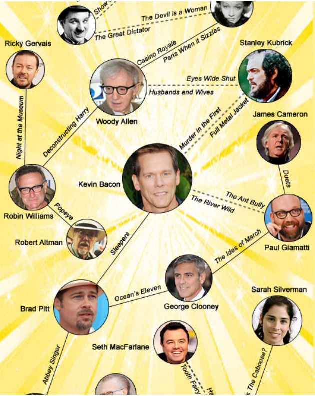


Definition of a Graph

- *Undirected graph* $G = (V, E)$: set V of nodes and set E of edges, where $E \subseteq V \times V$. Elements of E are unordered pairs.
  - Say that edge e is *incident* on u and on v.
  - Exactly one edge between any pair of nodes.
  - G contains no self loops, i.e., no edges of the form (u, u).



Definition of a Graph

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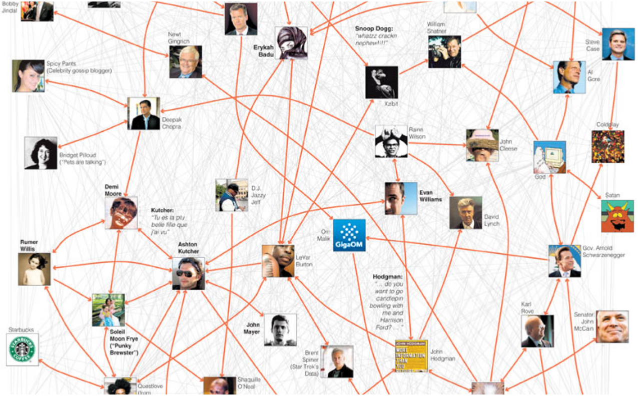
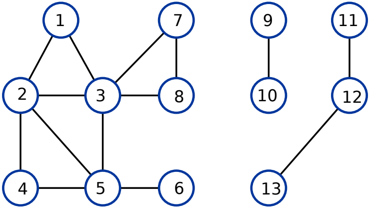
  - $e = (u, v)$: u is the *tail* of the edge e, v is its *head*; e is *directed from u to v*.
  - A pair of nodes $\{u, v\}$ may be connected by two directed edges: (u, v) and (v, u).
  - G contains no self loops.



Paths and Connectivity



- A *v_1-v_k path* in an undirected graph $G = (V, E)$ is a sequence P of nodes $v_1, v_2, \ldots, v_{k-1}, v_k \in V$ such that every consecutive pair of nodes $v_i, v_{i+1}, 1 \leq i < k$ is connected by an edge in E.

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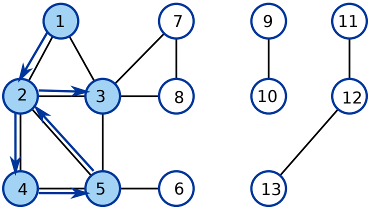

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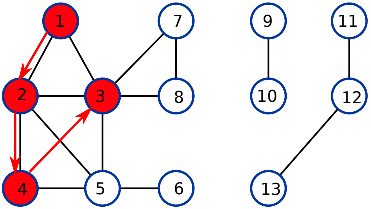

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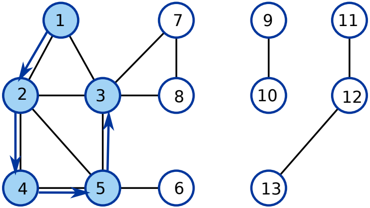

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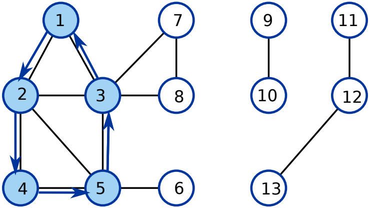

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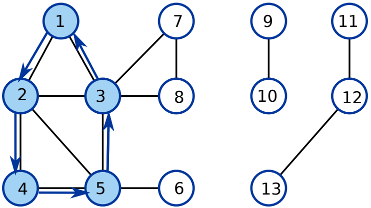

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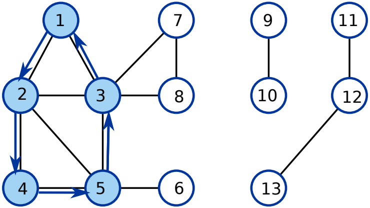

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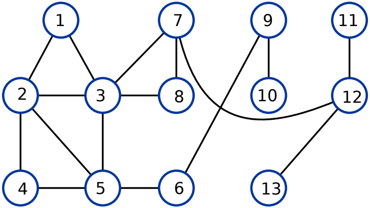

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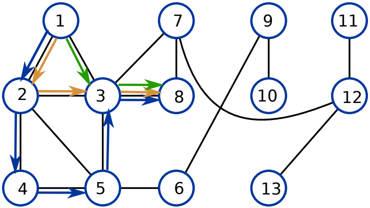

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- *Distance $d(u, v)$* between two nodes u and v is the minimum number of edges in any u-v path.

Trees

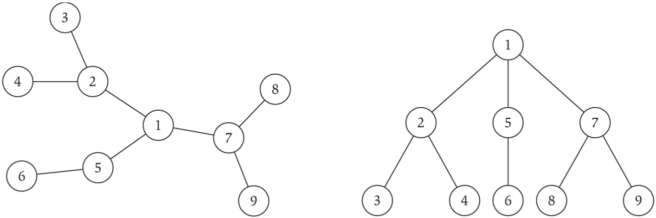

Figure 3.1 Two drawings of the same tree. On the right, the tree is rooted at node 1.

- An undirected graph is a *tree* if it is connected and does not contain a cycle.

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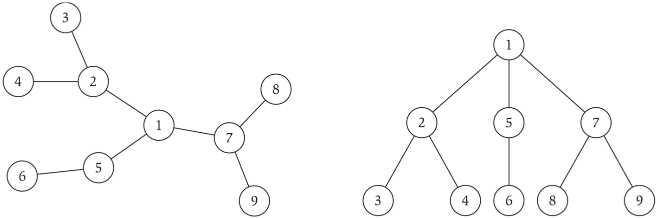

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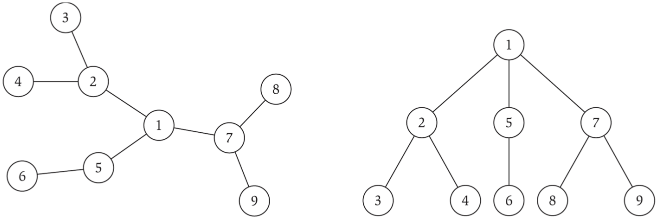

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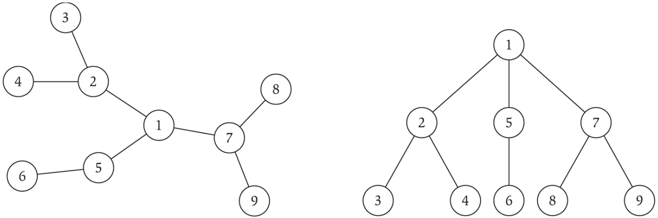

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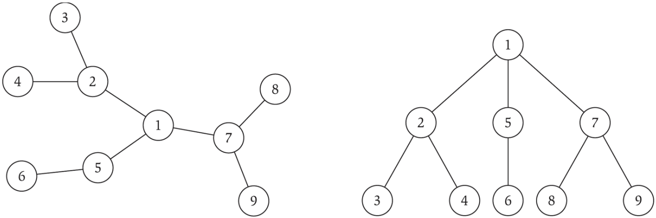

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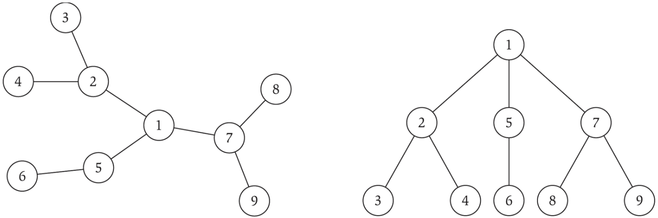

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- Examples of (rooted) trees: organisational hierarchy, class hierarchies in object-oriented languages.

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  3. G contains $n-1$ edges.

  - Note that none of these statements uses the word "tree".
  - 1 and 2 $\Rightarrow$ 3: just proved.
  - 2 and 3 $\Rightarrow$ 1:

Number of Edges in a Tree

- Claim: every n-node tree has exactly $n - 1$ edges.
- Proof 1: Root the tree. Each node, except the root, has a unique parent. Each edge connects one parent to one child. Therefore, the tree has $n - 1$ edges.
- Proof 2: (by induction) Two key pieces.
  - Every tree contains at least one leaf, i.e., node of degree 1. Why?
  - Inductive hypothesis: *every* tree with $n - 1$ nodes contains $n - 2$ edges.
- Stronger claim: Let G be an undirected graph on n nodes. Any two of the following statements implies the third:
  1. G is connected.
  2. G does not contain a cycle.
  3. G contains $n - 1$ edges.

  - Note that none of these statements uses the word "tree".
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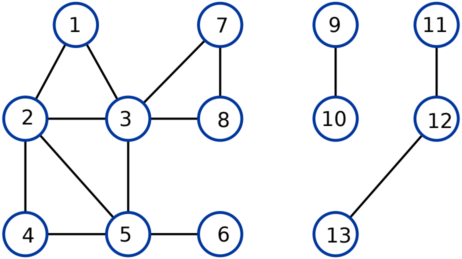
  - 2 and 3 $\Rightarrow$ 1: prove by contradiction.
  - 3 and 1 $\Rightarrow$ 2: prove yourself.

s-t Connectivity



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QUESTION: Is there an s-t path in G?

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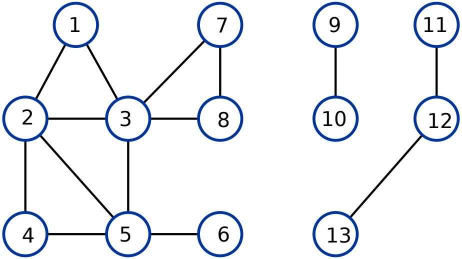

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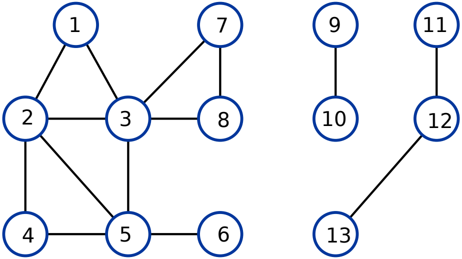

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- The *connected component of G containing s* is the set of all nodes u such that there is an s-u path in G.
- Algorithm for the s-t Connectivity problem: compute the connected component of G that contains s and check if t is in that component.

Computing Connected Components

- "Explore" G starting from s and maintain set R of visited nodes.

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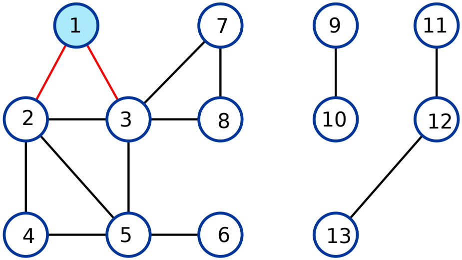

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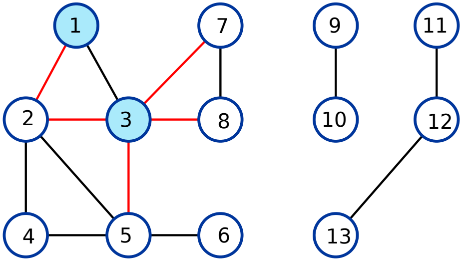

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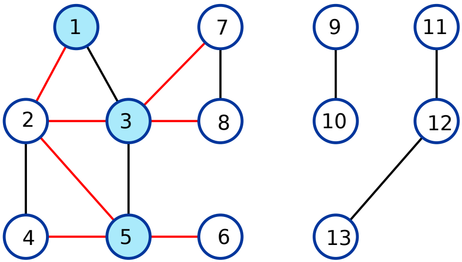

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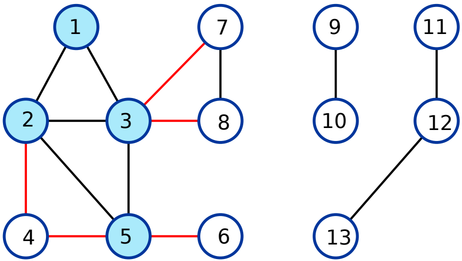

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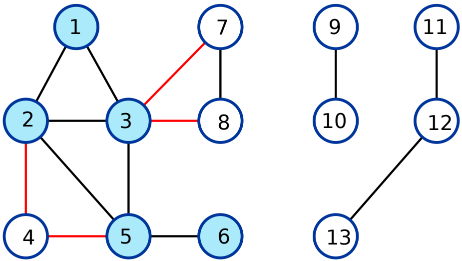

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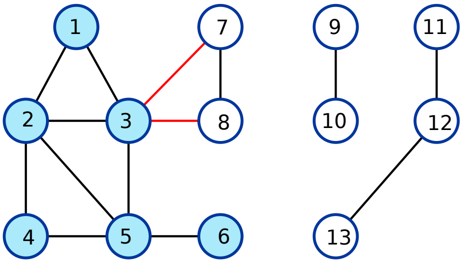

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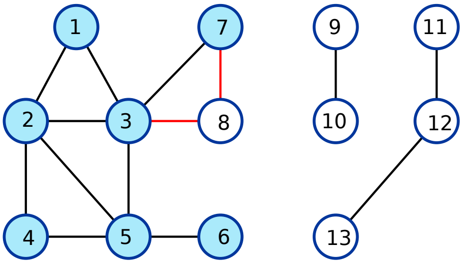

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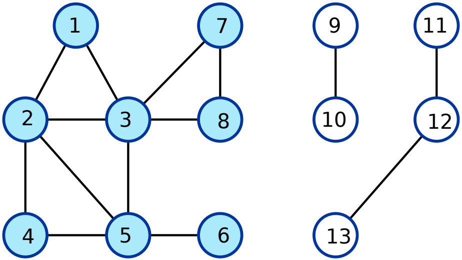

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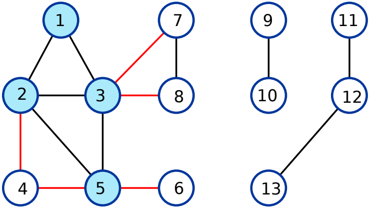

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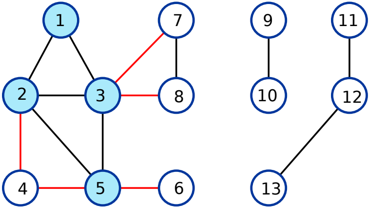

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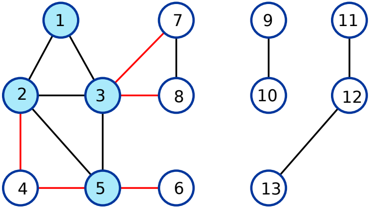

- How do we implement the `while` loop? Examine each edge in E.
- Other issues to consider:
  - Why does the algorithm terminate?
  - Does the algorithm truly compute connected component of G containing s?
  - What is the running time of the algorithm?

Termination of the Algorithm

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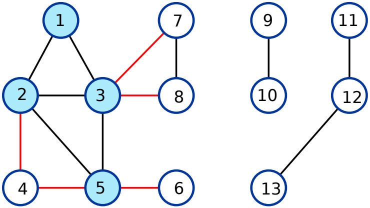

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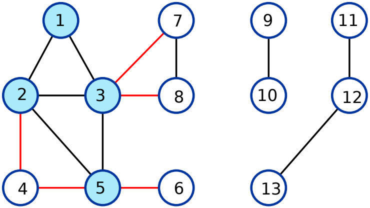

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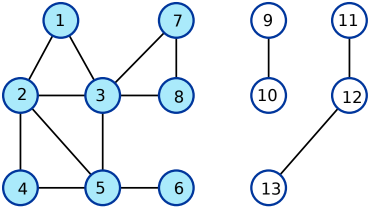

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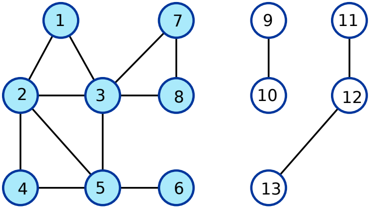

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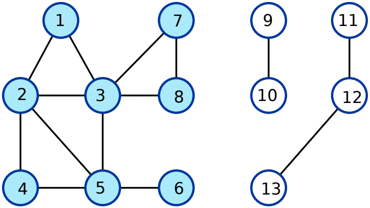

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  - either $R = V$ at the end or
  - in the last iteration, every edge either has both nodes in R or both nodes not in R.

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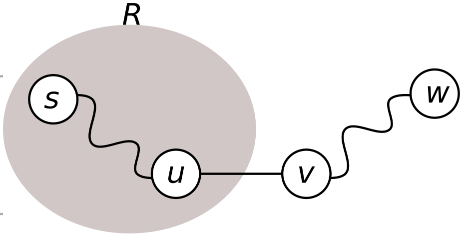

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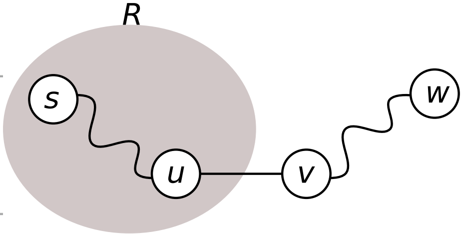

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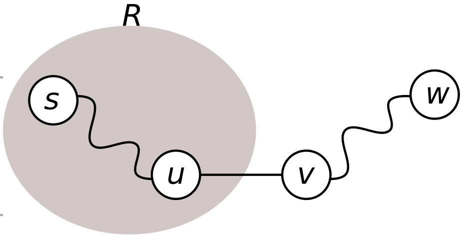

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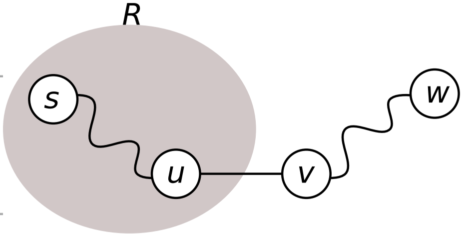

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  - Note: wrong to assume that predecessor of w in P is not in R.

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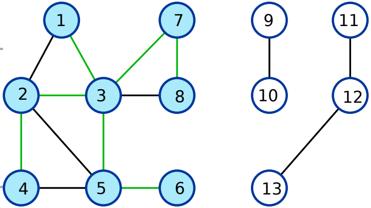

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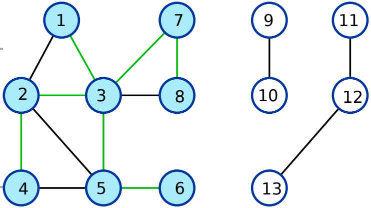

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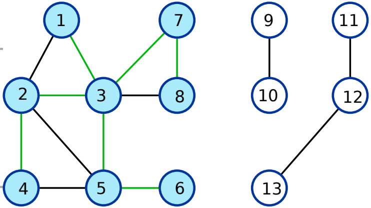

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- To recover the s-t path, trace these edges backwards from t until we reach s.

Running Time of the Algorithm

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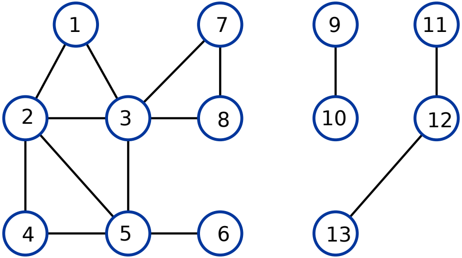
- Can we improve the running time by processing edges more carefully?

Breadth-First Search (BFS)



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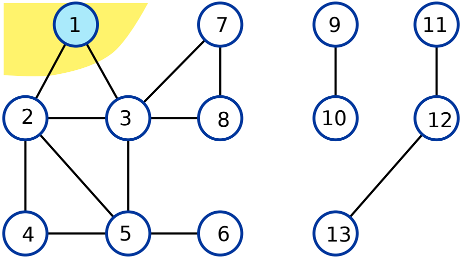

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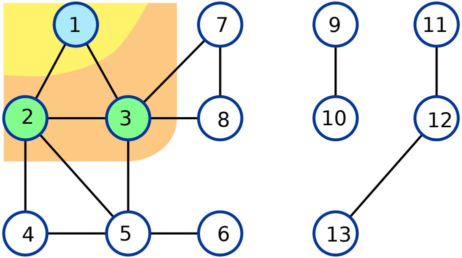

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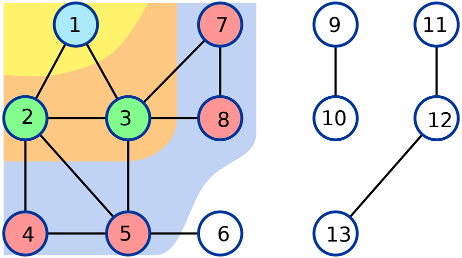

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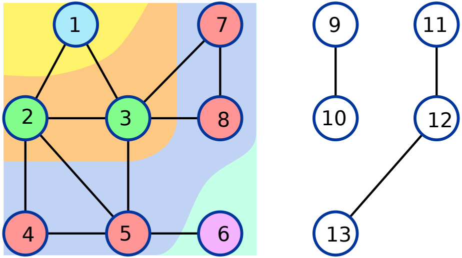

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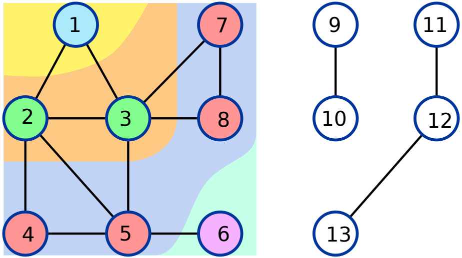
Properties of BFS



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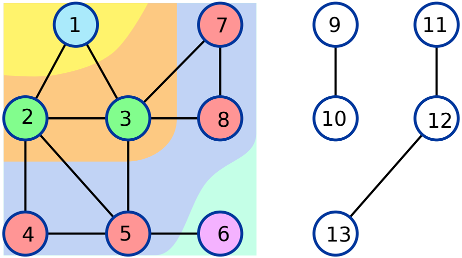

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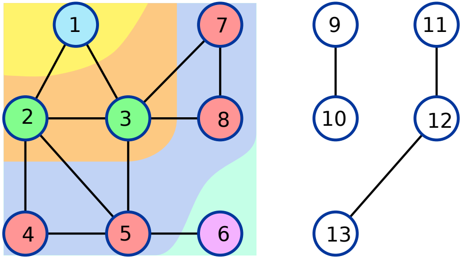

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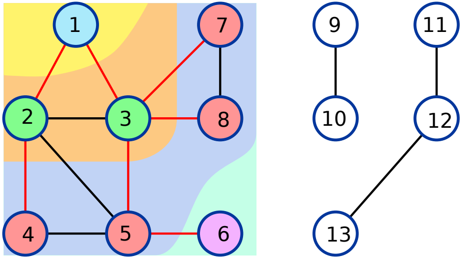

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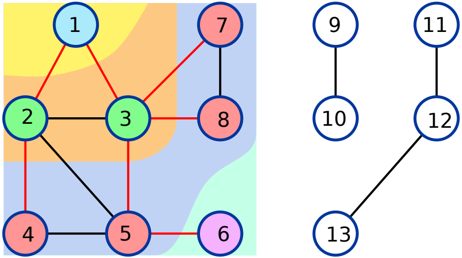

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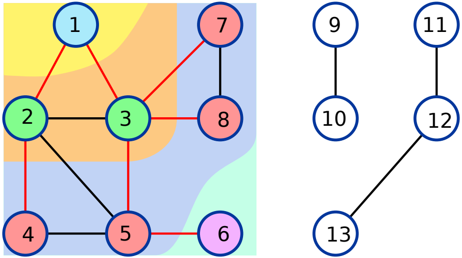

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  - T is called the *breadth-first search tree*.

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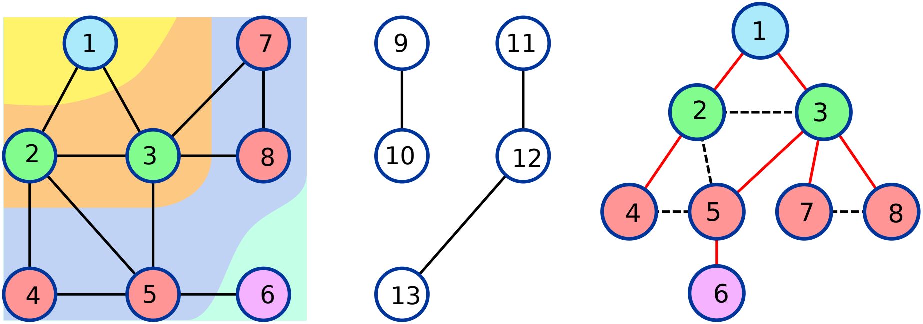

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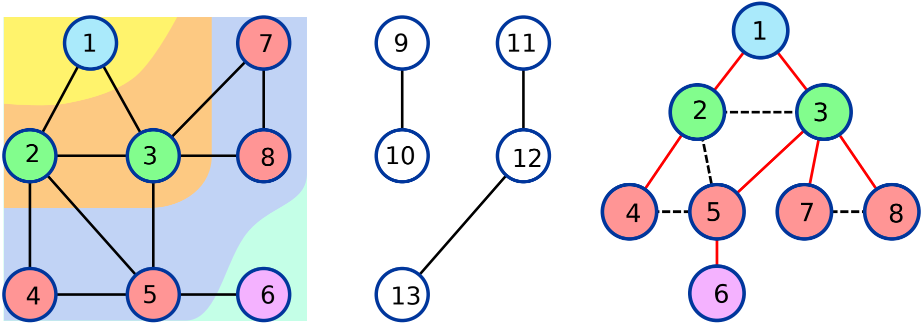

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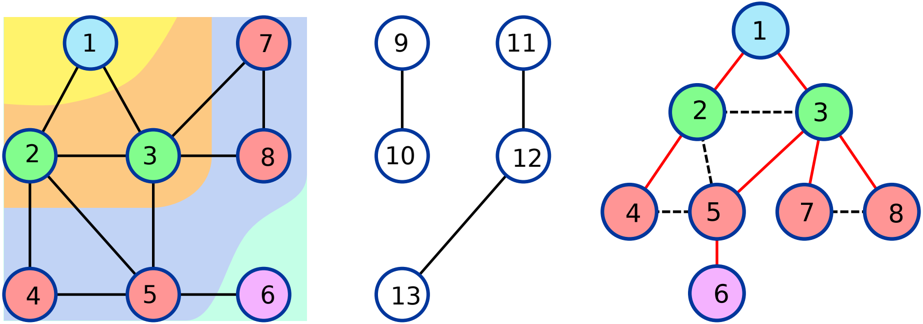

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- Still unresolved: an efficient implementation of BFS.

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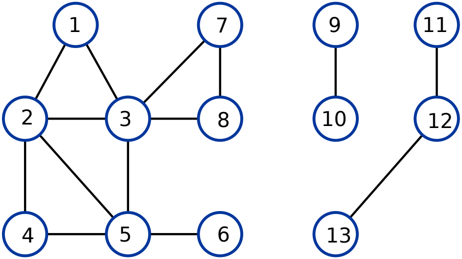
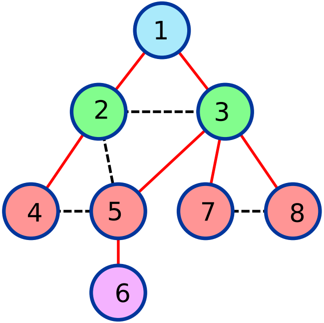
Example of DFS





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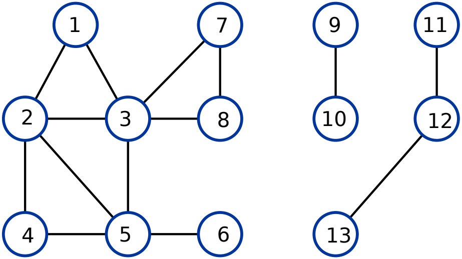

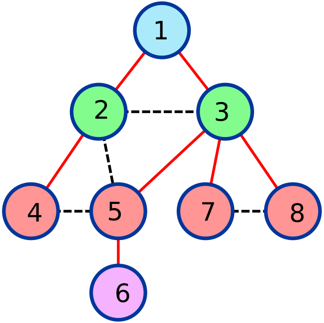

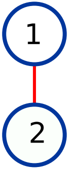

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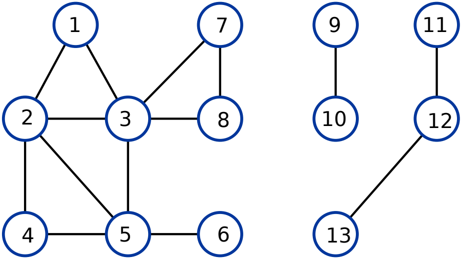

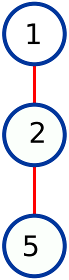

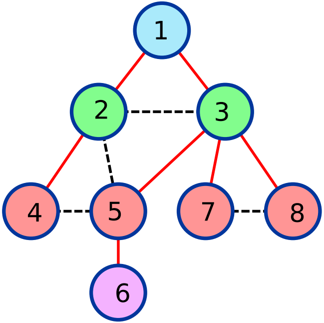

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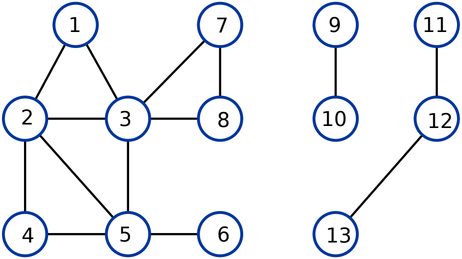

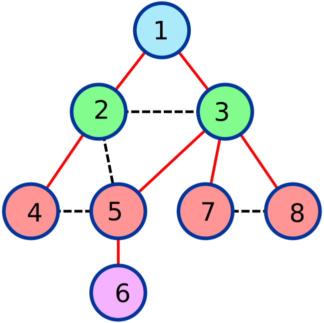

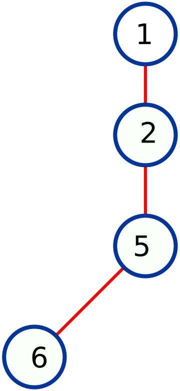

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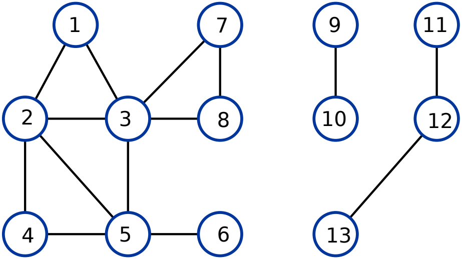

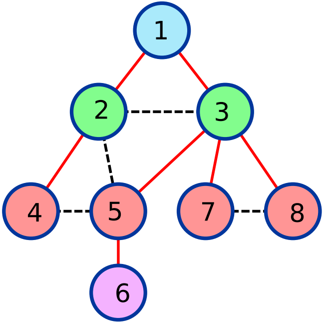

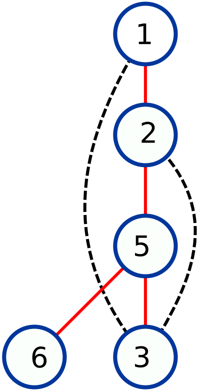

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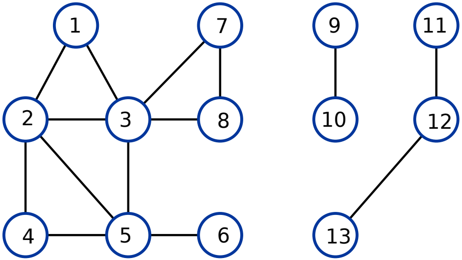

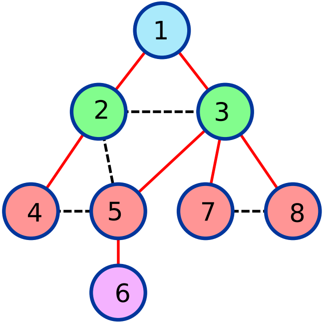

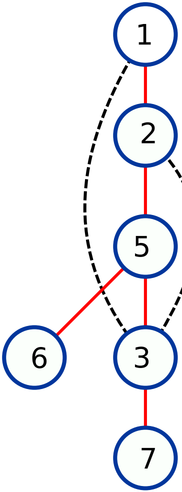

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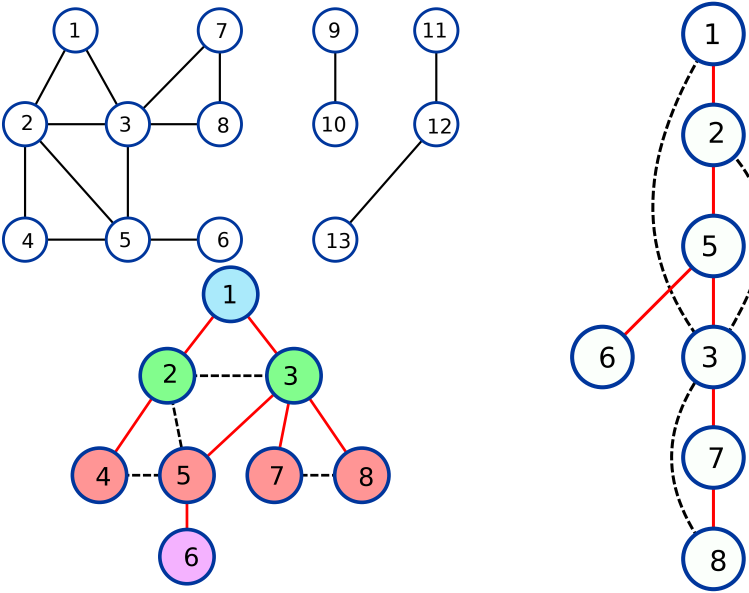

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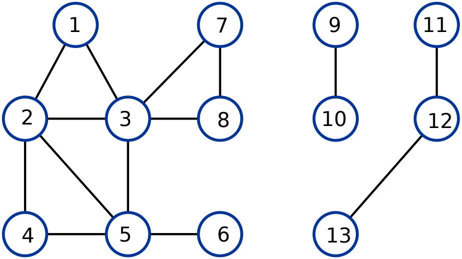

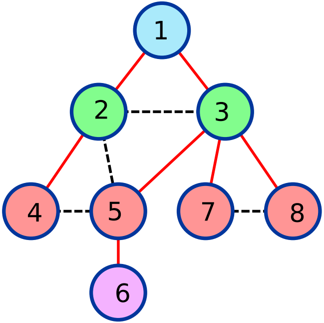

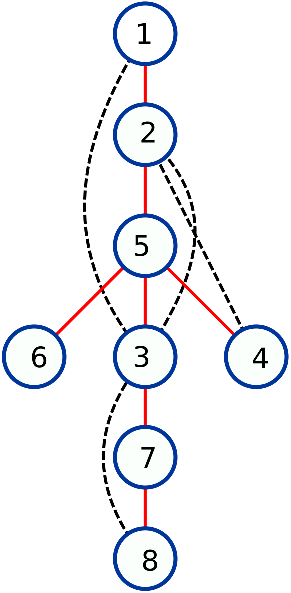

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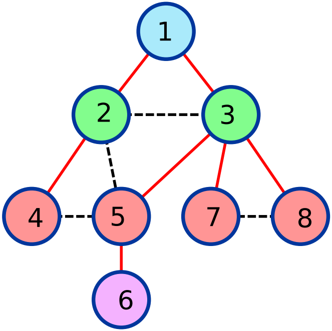

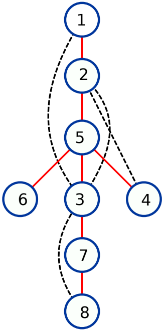

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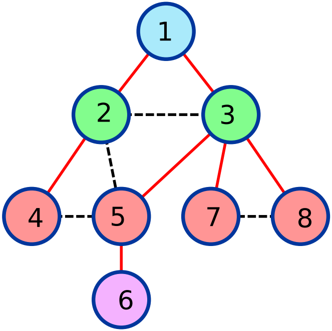

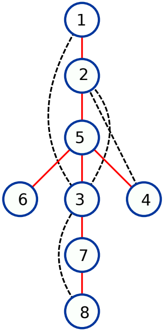

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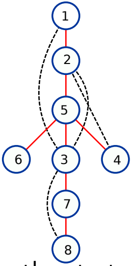

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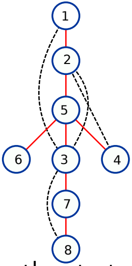

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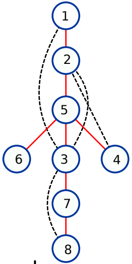

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- Proof: Assume, without loss of generality, that DFS(u) reached x first.
  - Since (x, y) is an edge in G, it is examined during DFS(x).
  - Since $(x, y) \notin T$, y must be marked as "Explored" during DFS(x) but before (x, y) is examined.
  - Since y was not marked as "Explored" before DFS(x) was invoked, it must be marked as "Explored" between the start and the end of DFS(x).
  - Therefore, y must be a descendant of x in T.

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Computing All Connected Components

1. Pick an arbitrary node s in G.
2. Compute its connected component using BFS (or DFS).
3. Find a node (say v, not already visited) and repeat the BFS from v.
4. Repeat this process until all nodes are visited.

Representing Graphs

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Representing Graphs

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Data Structures for Implementation

- "Implementation" of BFS and DFS: fully specify the algorithms and data structures so that we can obtain provably efficient times.
- Inner loop of both BFS and DFS: process the set of edges incident on a given node and the set of visited nodes.
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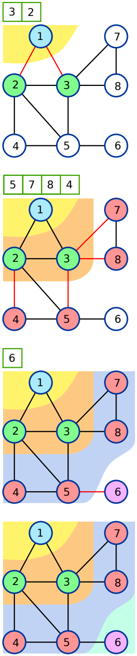
- How do we store the set of visited nodes? Order in which we process the nodes is crucial.
  - BFS: store visited nodes in a queue (first-in, first-out).
  - DFS: store visited nodes in a stack (last-in, first-out)

Implementing BFS

- Maintain an array `Discovered` and set `Discovered`$[v]$ = *true* as soon as the algorithm sees v.

```
BFS(s):
  Set Discovered[s] = true and Discovered[v] = false for all other v
  Initialize L[0] to consist of the single element s
  Set the layer counter i = 0
  Set the current BFS tree T = Ø
  While L[i] is not empty
    Initialize an empty list L[i+1]
    For each node u ∈ L[i]
      Consider each edge (u, v) incident to u
      If Discovered[v] = false then
        Set Discovered[v] = true
        Add edge (u, v) to the tree T
        Add v to the list L[i+1]
      Endif
    Endfor
    Increment the layer counter i by one
  Endwhile
```



Using a Queue in BFS

- Instead of storing each layer in a different list, maintain all the layers in a single queue L.

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BFS(s):
   Set Discovered[s] = true
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   Initialize L to consist of the single element s
   While L is not empty
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      Consider each edge (u, v) incident on u
      If Discovered[v] = false then
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         Add edge (u, v) to the tree T
         Push v to the back of L
      Endif
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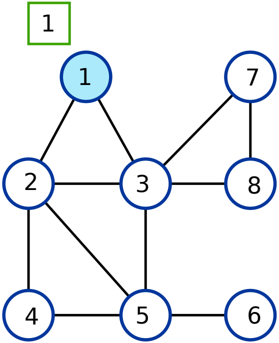

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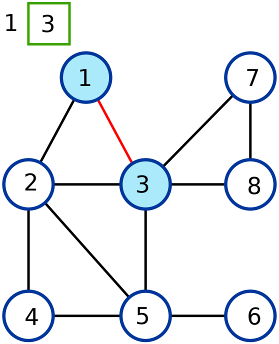

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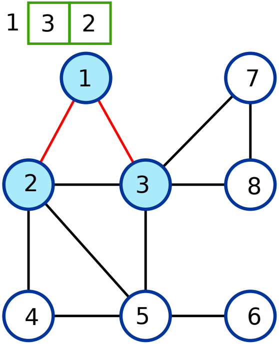

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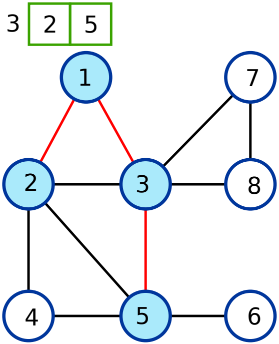

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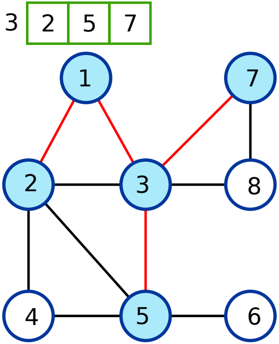

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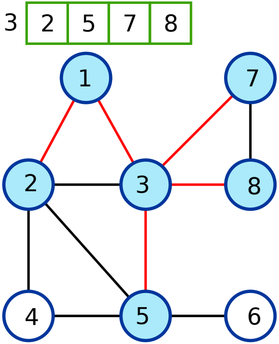

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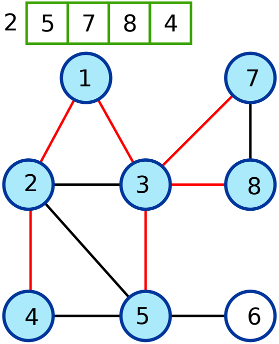

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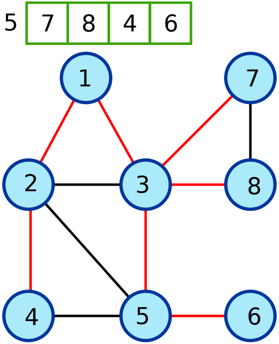

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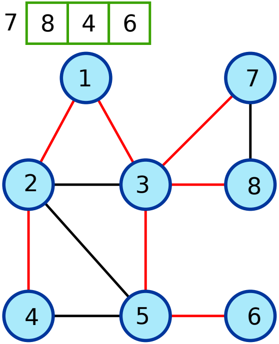

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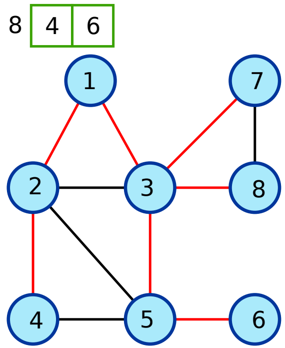

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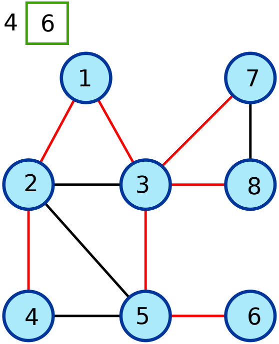

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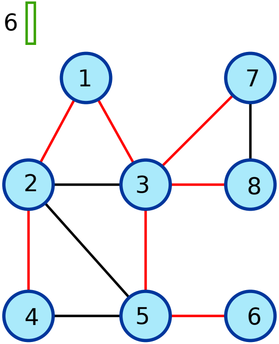

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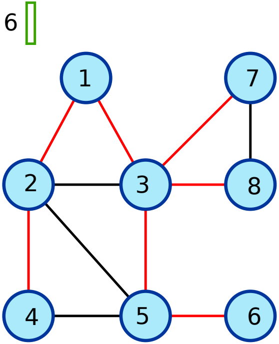

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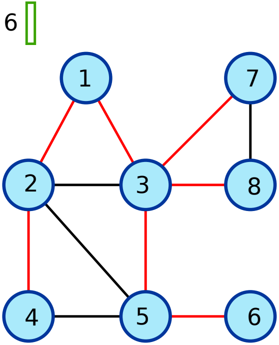

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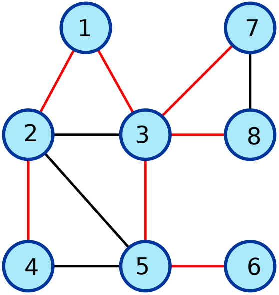

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- Claim: Nodes in layer $i+1$ will appear in L immediately after nodes in layer i.

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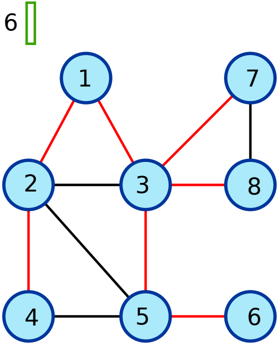

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  - Total time for all `for` loops: $\sum_{u \in G} O(n_u) = O(m)$ time.
  - Maintaining layer information:

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- Naive bound on running time is $O(n^2)$: For each node, we spend $O(n)$ time.
- Improved bound:
  - How many times is a node popped from L? Exactly once.
  - Time used by `for` loop for a node u: $O(n_u)$ time.
  - Total time for all `for` loops: $\sum_{u \in G} O(n_u) = O(m)$ time.
  - Maintaining layer information: $O(1)$ time per node.
  - Total time is $O(n + m)$.

Recursive DFS

```
DFS(u):
  Mark u as "Explored" and add u to R
  For each edge (u,v) incident to u
    If v is not marked "Explored" then
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- Procedure has “tail recursion”: recursive call is the last step.

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- Procedure has "tail recursion": recursive call is the last step.
- Can replace the recursion by an iteration: use a stack to explicitly implement the recursion.

Implementing DFS

- Maintain a stack S to store nodes to be explored.
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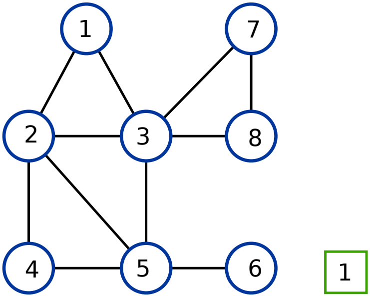

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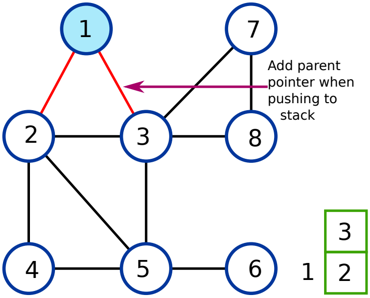

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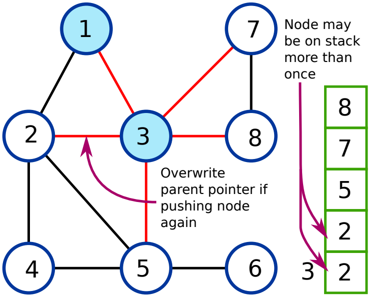

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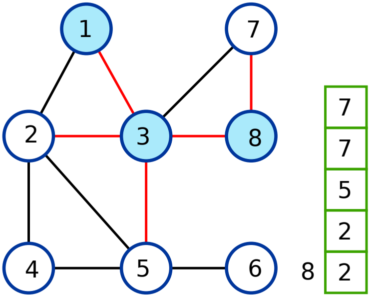

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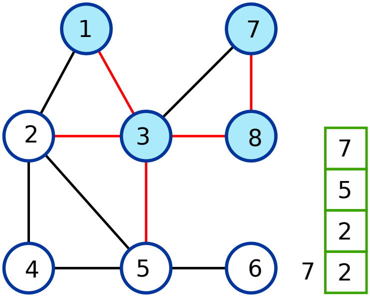

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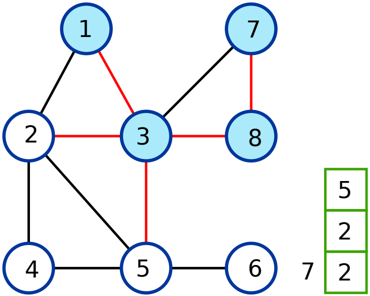

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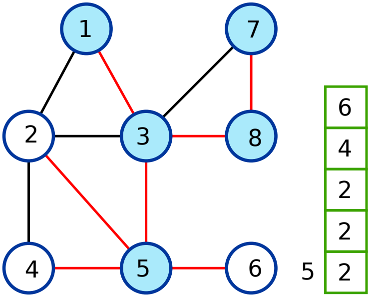

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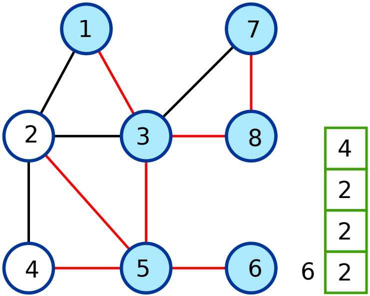

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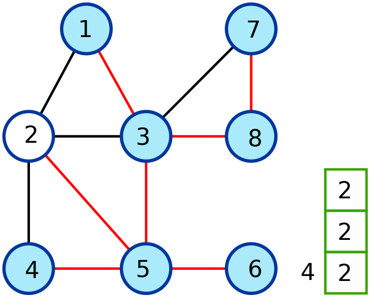

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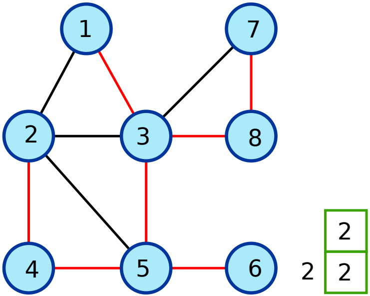

Comparing Recursion and Iteration

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- The total running time of the algorithm is $O(n + m)$.