# Homework 2

CS 5104 (Fall 2009)

Assigned on Wednesday, September 9, 2009.
Hardcopy due at the beginning of class on Wednesday, September 16, 2009.

**Instructions:**

- Do not forget to typeset your solutions. *Every mathematical expression must be typeset as a mathematical expression, e.g., the square of $n$ must appear as $n^2$ and not as "n^2".* Students using LaTeX or LyX can use the corresponding versions of the homework problems to start entering their solutions.
- Describe your algorithms as clearly as possible. The style used in the book is fine, as long as your description is not ambiguous.
- Do not make any assumptions not stated in the problem. If you do make any assumptions, state them clearly, and explain why the assumption does not decrease the generality of your solution.
- Do not describe your algorithms only for a specific example you may have worked out.
- You must also provide a clear proof that your solution is correct (or a counter-example, where applicable). Type out all the statements you need to complete your proof. *You must convince us that you can write out the complete proof. You will lose points if you work out some details of the proof in your head but do not type them out in your solution.*
- Analyse your algorithm and state the running time. You will only get partial credit if your analysis is not tight, i.e., if the bound you prove for your algorithm is not the best upper bound possible.

**Problem 1** (25 points) Solve exercise 2 in Chapter 3 (page 107) of your textbook. Your proof of correctness must consider two cases:

(a) The graph does not contain a cycle: in this case, you should prove that your algorithm correctly reports this fact.

(b) The graph contains one or more cycles: in this case, you must prove that the algorithm correctly computes one of the cycles in the graph. If the graph contains many cycles, it is enough to report one of the cycles.

**Problem 2** (25 points) Solve exercise 5 in Chapter 3 (page 108) of your textbook. Given a tree $T$, let us define two useful quantities: $c_T$ ($c$ with $T$ as a subscript) the number of nodes in $T$ with two children, and $l_T$ ($l$ with $T$ as a subscript) the number of leaves in $T$. With these two quantities defined, the goal of the problem is to prove that for every tree $T$, $c_T = l_T - 1$. To assist you with the proof, here are three candidates for the induction hypothesis. In your solution, state which candidate is correct and then provide the complete proof by induction based on this choice.

(i) There exists a tree $T$ on $n-1$ nodes such that $c_T = l_T - 1$.

(ii) For every integer $k$ between 1 and $n-1$, there exists a tree $T$ on $k$ nodes such that $c_T = l_T - 1$.

(iii) For every tree $T$ on $n-1$ nodes, $c_T = l_T - 1$.

**Problem 3** (20 points) Solve exercise 7 in Chapter 3 (page 108-109) of your textbook.

**Problem 4** (30 points) Solve exercise 9 in Chapter 3 (page 110) of your textbook. *Hint:* Many of the layers in the BFS tree rooted at $s$ have a special property.