CS3414. Homework problem set II. **10 points per problem**, unless otherwise stated. Do not submit any codes, but be prepared to email one at TA/instructor's request.

C&K = *Cheney and Kincaid* textbook. You can write your codes in C or C++. Do not submit the code unless asked for. Instead, outline your solution by showing the key steps in the algorithm used.

1. C&K 26, page 32.
2. C&K 10, page 32.
3. Use Taylor series to show that the *truncation* error involved in calculating of the derivative $f'(x) \approx (f(x+h/2) - f(x-h/2))/h$ is of order h^2, *i.e.* (truncation) $error = A * h^2$.

3a. Use the above result, and the *round-off error* estimate discussed in class, to derive an expression for the *total* error involved in calculating $f'(x)$. For simplicity, assume $f'(x) \sim 1$, along with its derivatives.

Find an estimate of the optimal step h that minimizes that error as a function of ϵ_{mach}. For your laptop (double precision), what is h and the associated total error? How does it compare with the optimal h and the error for the formula discussed in class, $f'(x) \approx (f(x+h) - f(x))/h$?

4. C&K 1, page 63
5. Write an *efficient* code that computes $exp(x)$ for any $-25 < x < 25$ to within 3 decimal points. Provide printouts for $x = 0.1, +20, -20$. Clearly indicate which algorithms are used for different values of x.
6. Write a code that produces accurate (within machine precision) values of $f(x) = \frac{(x - sin(x))}{x^3}$ for $0 < x < 1$. Print out results for $x = 0.5, 10^{-16}$. Clearly indicate which algorithms are used for the two different values of x. In this problem, you can use $sin(x)$ function supplied by standard libraries.
7. Use `Series` command in *Mathematica* to find first 6 terms in the Taylor series expansion of $cos(x)$. Given that, how would you compute $cos(10.0)$? (10 rad).