

**Problem I, 10 points.**

Modify `numderivative.cc` to calculate the derivative  $\exp(x)'$  at  $x = 1$  to within 0.05 % relative error by using:  $f'(x) \approx (f(x + h/2) - f(x - h/2))/h$ . What step size  $h$  will you need? What is an advantage of the above formula compared to the one we used in class [ *i.e.*  $f'(x) \approx (f(x + h) - f(x))/h$  ]? Using `numderivative.cc`, find the optimum  $h$ , and compare it to the optimum for the original `numderivative.cc`.

**Problem II, 20 points.**

In `numderivative.cc` replace the “`exp(x)`” with “`sin(1/x)`” where appropriate to obtain a numerical estimate for the derivative of  $f(x) = \sin(1/x)$  at

**a)**  $x = 1/\pi$ . Choose “`h`” so that the result is accurate to within at least 4 decimal points. What is your calculated result?

**b)** What happens when you try the same code for  $x = 10^{-20}/\pi$ ? Why? Use the chain rule to re-formulate the problem into a mathematically equivalent one that is free from the defect, modify the code, and re-compute. What do you get now?