CS3414. Example problems.

Question I

Calculating first derivative *approximately* via $f'(x) \approx \frac{[f(x+h)-f(x-h)]}{2h}$ results in a *truncation error* $error_{tr} \sim h^2$. The *round-off error* involved is of an order of $error_{roff} = \frac{\epsilon_{mach}}{h}$. An optimal value of the step-size h should minimize $error_{tr} + error_{roff}$. What (order-of-magnitude) would that optimal h be on a computer with $\epsilon_{mach} = 10^{-18}$

Question II

A student who did not take CS3414 is attempting to use Newton's method to solve $sin^3(x)e^{-x^2} = 0$. His code prints out a sequence of differences $Q_n = |x_{n+1} - x_n|$, where x_n is the n^{th} approximation to the solution (x_0 is the initial guess). The first five Q_n (for n = 0, 1, 2, 3, 4) are: 0.1, 0.52, 1.11, 0.47, 1.14, 0.51, 1.13.... Your recommendation to the student may be one of the following EXCEPT:

A) Use a different x_0 B) Use *bisection* method instead. C) Continue for 50 more steps. D) Solve the problem analytically.

Question III

You are looking for a local minimum of a function $f(x)$ which lies somewhere between 0 and 2. You have chosen the *golden section* method and you want to locate the minimum to within $TOL = 10^{-3}$ from the exact value (that is when you have stopped, $|x_n^{approx} - x_{min}^{exact}| < TOL$ Roughly how many iterations n will you need?

Question IV

You are looking for a local minimum of a function $f(x)$, $1 < x < 2$, and you have chosen the *Newton's method* . You want to locate the minimum as accurately as you can on a machine with $\epsilon_{mach} = 10^{-16}$. Roughly how many iterations n you may need, assuming that you have made a good initial guess x_0^{approx}, $|x_0^{approx} - x_{min}^{exact}| \approx 0.1$, and the method is converging at its optimal speed?

(A) 100 (B) 30 (C) 10 (D) 5

Question V

Your "numerical Methods" professor has told you that he developed a new minimiation method that is *guaranteed* to find global minimum of any 500-dimensional continuous function, on a laptop, with 24 hrs. Circle all statements below that are likely true.

(A) He is pulling your leg. (B) The method works, but only on quadratic functions such as $f(x) = \sum_{i=1}^{500} x_i^2$ (C) While the method fails for some 500-dimensional functions, it works perfectly on any 100-dimensional one. (D) The method works as described, it uses a modification of "steepest descent" (gradient) approach with half the optimal step. (E) The method can find many local minima.