## Problem I, 10 points.

Modify numderivative.cc to calculate the derivative exp(x)' at x=1 to within 0.05 % relative error by using:  $f'(x) \approx (f(x+h/2)-f(x-h/2))/h$ . What step size h will you need? What is an advantage of the above formula compared to the one we used in class [ i.e.  $f'(x) \approx (f(x+h)-f(x))/h$  ]? Why does it have this advantage? Using numderivative.cc , find the optimum h (the one that minimizes the relative error), and compare it to the optimum for the original numderivative.cc .

## Problem II, 20 points.

In the original numderivative.cc replace the " $\exp(x)$ " with " $\sin(1/x)$ " where appropriate to obtain a numerical estimate for the derivative of  $f(x) = \sin(1/x)$  at

- a)  $x = 1/\pi$ . Choose "h" so that the result is accurate to within at least 4 decimal points. What is your calculated result?
- **b)** What happens when you try the same code for  $x = 10^{-20}/\pi$ ? Why? Use the chain rule to re-formulate the problem into a mathematically equivalent one that is free from the defect, modify the code, and re-compute. What do you get now?