CS3414. EXAMPLE PROBLEMS. SOLVING ODEs

1 Question

You are solving several ODEs together; each one looks like $y_i'' = k_i * y_i$, where $k_i < 0$ for all i. This ODE is known to have the exact solution: $y_i(t) = A * sin(\omega_i t + \phi_i)$, where ϕ_i is determined by the initial condition at $t = 0$.

You have chosen the 1st order Euler method discussed in class to solve the problem numerically, with one and the same step size h for all the ODEs.

1. (5 points) What is reasonable guess for the step size h? (circle the one correct answer). `EXPLAIN.`

   A) $h \ll minimum\ of\ \{1/\sqrt{k_i}\}$

   B) $h \ll maximum\ of\ \{1/\sqrt{k_i}\}$

   C) $h \gg minimum\ of\ \{1/\sqrt{k_i}\}$

   D) $h \gg maximum\ of\ \{1/\sqrt{k_i}\}$

   E) $h = 0.1 * \epsilon_{mach}$

   F) $h = \epsilon_{mach}$

   SOLUTION:

2. (5 points) Using Euler's method you quickly find a numerical solution to the above ODE, which has these points in it: t(1.8) = 0.1; t(3.14) = 1; t(10) = 7.1; t(100) = -723.1; What is a likely explanation for the pattern? `EXPLAIN.`

   (A) The original ODE is unstable. (B) The step-size chosen is too large. (C) The step side is too small.

   SOLUTION.

2 Question

You are solving $y' = y^2$, $y(0) = 1$ using a numerical method, say Euler's. You are finding that no matter how small a step size h you take, you just can not obtain an accurate solution within a seemingly small range $0 < t < 1$. EXPLAIN.

SOLUTION: