Question I

Calculating first derivative approximately via \( f'(x) \approx \frac{[f(x+h)-f(x-h)]}{2h} \) results in a truncation error \( \text{error}_{tr} \sim h^2 \). The round-off error involved is of an order of \( \text{error}_{roff} = \frac{\epsilon_{\text{mach}}}{h} \). An optimal value of the step-size \( h \) should minimize \( \text{error}_{tr} + \text{error}_{roff} \). What (order-of-magnitude) would that optimal \( h \) be on a computer with \( \epsilon_{\text{mach}} = 10^{-18} \)

Question II

A student who did not take CS3414 is attempting to use Newton’s method to solve \( \sin^3(x)e^{-x^2} = 0 \). His code prints out a sequence of differences \( Q_n = |x_{n+1} - x_n| \), where \( x_n \) is the \( n^{th} \) approximation to the solution (\( x_0 \) is the initial guess). The first five \( Q_n \) (for \( n = 0, 1, 2, 3, 4 \)) are: 0.1, 0.52, 1.11, 0.47, 1.14, 0.51, 1.13.... Your recommendation to the student may be one of the following EXCEPT:

A) Use a different \( x_0 \)  
B) Use bisection method instead.  
C) Continue for 50 more steps.  
D) Solve the problem analytically.

Question III

You are looking for a local minimum of a function \( f(x) \) which lies somewhere between 0 and 2. You have chosen the golden section method and you want to locate the minimum to within \( TOL = 10^{-3} \) from the exact value (that is when you have stopped, \( |x_{\text{approx}}^n - x_{\text{exact}}^n| < TOL \)). Roughly how many iterations \( n \) will you need?

Question IV

You are looking for a local minimum of a function \( f(x) \), \( 1 < x < 2 \), and you have chosen the Newton’s method. You want to locate the minimum as accurately as you can on a machine with \( \epsilon_{\text{mach}} = 10^{-16} \). Roughly how many iterations \( n \) you may need, assuming that you have made a good initial guess \( x_{\text{approx}}^0 \), \( |x_{\text{approx}}^0 - x_{\text{exact}}^0| \approx 0.1 \), and the method is converging at its optimal speed?

(A) 100  
(B) 30  
(C) 10  
(D) 5

Question V

Your "numerical Methods” professor has told you that he developed a new minimization method that is guaranteed to find global minimum of any 500-dimensional continuous function, on a laptop, with 24 hrs. Circle all statements below that are likely true.

(A) He is pulling your leg.  
(B) The method works, but only on quadratic functions such as \( f(x) = \sum_{i=1}^{500} x_i^2 \)  
(C) While the method fails for some 500-dimensional functions, it works perfectly on any 100-dimensional one.  
(D) The method works as described, it uses a modification of ”steepest descent” (gradient) approach with half the optimal step.  
(E) The method can find many local minima.