

B. Simple quadrature formulas

1. Midpoint rule

- Basic rule

$$f(m)(b - a), \text{ where } m = \frac{a + b}{2}$$

- Composite rule

$$\sum_{i=0}^{n-1} f\left(\frac{x_{i+1} + x_i}{2}\right) (x_{i+1} - x_i) + O(h^2), \text{ where } h \approx (x_{i+1} - x_i)$$

2. Trapezoid rule

- Basic rule

$$\frac{b - a}{2} (f(a) + f(b))$$

- Composite rule

$$h \left(\frac{1}{2} f(x_0) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(x_n) \right) - \frac{(b - a) f''(c)}{12} h^2$$

3. Simpson's rule

- Basic rule

$$\frac{b - a}{6} (f(a) + 4f(m) + f(b))$$

- Composite rule

$$\frac{h}{3} \left(f(x_0) + 4 \sum_{j=1}^s f(x_{2j-1}) + 2 \sum_{j=1}^{s-1} f(x_{2j}) + f(x_n) \right) + O(h^4), \text{ where } n = 2s$$