You will submit your solution to this assignment to the Curator System (as HW02). Your solution must be either a plain text file (e.g., NotePad) or a typed MS Word document; submissions in other formats will not be graded.

Except as noted, credit will only be given if you show relevant work.

1. [15 points] Using the rules given in the course notes, perform an exact count complexity analysis, for the worst case, of the body of the following function.

   ```java
   public double eval(double[] c, double x) {
       double polyx = c[0];
       for (int k = 1; k < c.length; k++) {
           double xToK = x;
           for (int i = 1; i < k; i++) {
               xToK = x * xToK;
           }
           polyx = polyx + c[k] * xToK;
       }
       return polyx;
   }
   ```

   State both a complexity function $T(N)$ and the $\Theta$-complexity of $T(N)$.

2. [15 points] Using the rules given in the course notes, perform an exact count complexity analysis, for the worst case, of the body of the following function.

   ```java
   public double eval(double[] c, double x) {
       double polyx = c[0];
       double xToK = x;
       for (int k = 1; k < c.length; k++) {
           polyx = polyx + c[k] * xToK;
           xToK = x * xToK;
       }
       return polyx;
   }
   ```

   State both a complexity function $T(N)$ and the $\Theta$-complexity of $T(N)$.

3. [20 points] For each part, determine the simplest possible function $g(n)$ such that the given function is $\Theta(g)$. No justification is necessary, but you might have to do some analysis using the theorems from the notes.

   a) $a(n) = 14n^2 + 3n \log n$

   b) $b(n) = 3n^2 \log n$

   c) $c(n) = 3n \log^2 n + 3n^2 \log n$

   d) $d(n) = 10n^2 + 2^n$

   e) $e(n) = \frac{n^2 + 2n + 3}{n}$
4. [15 points] Suppose that executing an algorithm on input of size \( N \) requires executing \( T(N) = N \log N + 16N \) instructions. How long would it take to execute this algorithm on hardware capable of carrying out \( 2^{32} \) instructions per second if \( N = 2^{30} \)? (Give your answer in hours, minutes and seconds, to the nearest second.)

5. [25 points] Design an efficient algorithm for solving the following problem:

Given an array \( A \) holding \( N \) elements, such that \( A[0] < A[1] < A[2] < \ldots < A[N-1] \), determine whether there is an index \( k \) such that \( 0 \leq k \leq N-1 \) and \( A[k] = k \).

Write your algorithm as a Java function and state its \( \Theta \)-complexity.

6. [10 points] Prove the following:

\[
\text{if } x \text{ is a real number then } \left\lfloor x \right\rfloor + 1 = \left\lfloor x + 1 \right\rfloor
\]