1. The following algorithm is the most obvious (but far from most efficient) way to evaluate a polynomial $f(x) = \sum_{i=0}^{N} a_i x^i$.

Assume that the coefficients are stored in an array $a[]$.

```c
Poly = 0;
for (i = 0; i <= N; i++) {
    xtoN = 1;
    for (j = 1; j <= i; j++)
        xtoN = x * xtoN;
    Poly = Poly + xtoN * a[i];
}
```

a) [20 points] Derive a complexity function $T(N)$ for the given algorithm.
b) [6 points] State the $\Theta$-complexity of $T(N)$.

2. [20 points] Consider the following function, where $\alpha$ is an unknown constant:

$$f(n) = n^\alpha + \log n$$

Use Theorem 8 from the course notes on asymptotics to prove the following fact:

$$\text{if } \alpha > 0, f(n) \text{ is } \Theta(n^\alpha)$$

Show all work to support every step in your proof. Hint: if $\alpha > 0$, then

$$\lim_{n \to \infty} n^\alpha = \infty$$

3. [24 points] Suppose that executing an algorithm on input of size $N$ requires executing $T(N) = 5N \log N + 8N$ instructions.

a) How long would it take to execute this algorithm on hardware capable of carrying out $2^{20}$ instructions per second if $N = 2^{24}$? (Give your answer in hours, minutes and seconds, to the nearest second.)

b) Repeat assuming the hardware can execute $2^{22}$ instructions per second.
4. [30 points] Divide the following functions into categories, so that two functions, say \( f(n) \) and \( g(n) \), are in the same category if and only if \( f(n) \) is \( \Theta(g(n)) \). Arrange the categories from the lowest order of magnitude to the highest. A function may be in a category by itself, or there may be several functions in the same category.

<table>
<thead>
<tr>
<th></th>
<th>( \log^5 n ) (i.e., ((\log n)^5))</th>
<th>( 3^n )</th>
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<tbody>
<tr>
<td>5000</td>
<td>( \log n )</td>
<td>( n^3 )</td>
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<td>( \log n )</td>
<td>( n + \log n )</td>
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<tr>
<td>( n^2 \log n )</td>
<td>( n^2 - 100n )</td>
<td>( 4n + n^{1/2} )</td>
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<td>( \log(\log n^2) )</td>
<td>( n^{0.3} )</td>
<td>( n^2 )</td>
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<td>( \log n^2 )</td>
<td>( (n^2 + 4)^{1/2} )</td>
<td>( 2^n )</td>
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