CS 2604 Homework 4 Fall 2004
Assigned: Thurs, Nov 18 Due: in-class at 2:00pm, Thurs, Dec 2

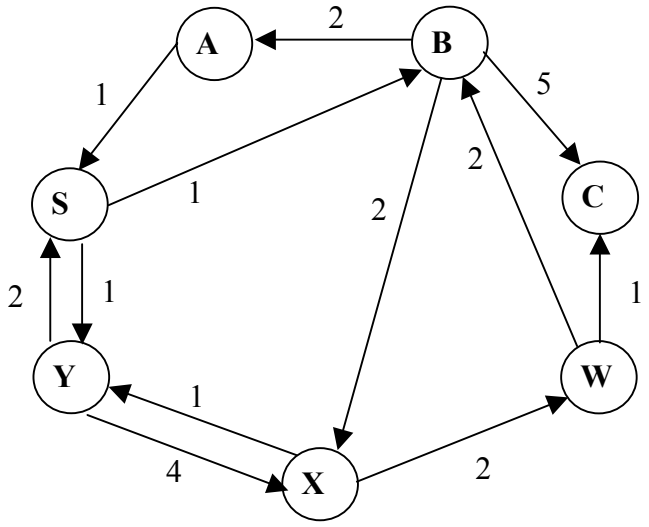
Name: _____

VT ID # _____

This homework assignment will be **handwritten and turned in in-class, at the start of class (2:00pm) on Thursday, December 2, 2004**. Write your answers on this document in the space provided for each question. Be sure to write your name on each page and keep the pages stapled together. No late assignments will be accepted.

For Problems 1-5, consider the following directed graph, G:

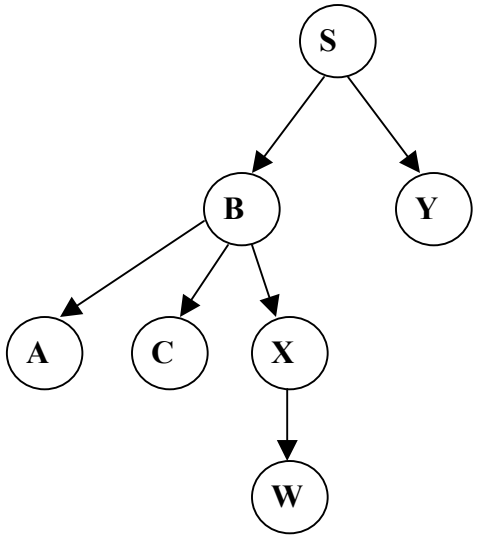
Here is a graphical representation of the graph G:



And here is an adjacency matrix representation of the same graph G:

	S	A	B	C	W	X	Y
S	–	–	1	–	–	–	1
A	1	–	–	–	–	–	–
B	–	2	–	5	–	2	–
C	–	–	–	–	–	–	–
W	–	–	2	1	–	–	–
X	–	–	–	–	2	–	1
Y	2	–	–	–	–	4	–

1. [15 points] Draw a graphical representation of the **breadth-first spanning tree** (the predecessor subgraph) for graph G. Use vertex S as the starting vertex. Assume that: 1) graph G is stored using an adjacency list representation, and that 2) each adjacency list is sorted in alphabetical order based on the letters shown in the vertices.



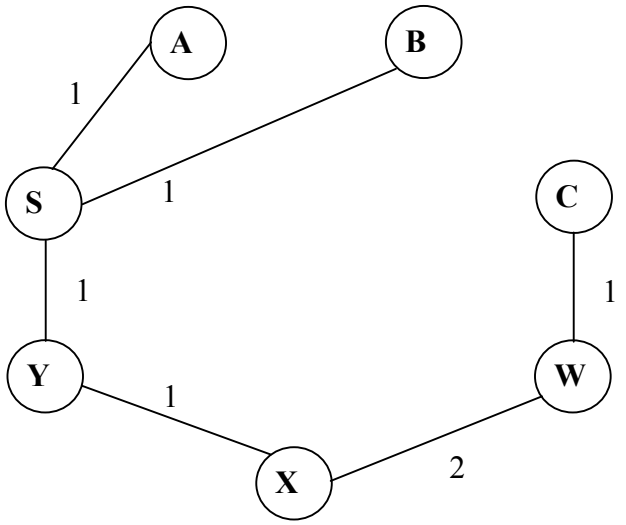
2. [15 points] Compute the **depth-first spanning forest** (the predecessor subgraph) for graph G. Use vertex S as the starting vertex. Use the depth-first traversal technique shown in class that increments a time counter by one each time a vertex is discovered or finished. **Express your answer by filling in the table below** with the discovery time, finishing time, and predecessor for each vertex. Several cells in the table have been filled in with the correct values to help get you started. Assume that: 1) graph G is stored using an adjacency list representation, and that 2) each adjacency list is sorted in alphabetical order based on the letters shown in the vertices.

	S	A	B	C	W	X	Y
Discovery Time	**0**	**2**	**1**	**4**	**7**	**6**	**9**
Finishing Time	**13**	**3**	**12**	**5**	**8**	**11**	**10**
Predecessor	**nil**	**B**	**S**	**B**	**X**	**B**	**X**

3. [10 points] Is it possible to give a topological sort of graph G? If so, give a topologically sorted ordering of the vertices. If not, briefly explain why.

It is NOT possible to give a topological sort of graph G because it contains cycles. Topological sorting can only be performed on directed acyclic graphs (DAGs).

4. [10 points] Draw a graphical representation of a minimum spanning tree (MST) of graph G. For this problem only, you should consider the edges of graph G as UNDIRECTED edges. In cases where this results in two edges between two vertices, use only the edge with the smaller weight. Note that more than one MST of G may exist. For this problem, you only need to show ONE MST of G.



One MST of G is shown above. There is one other:

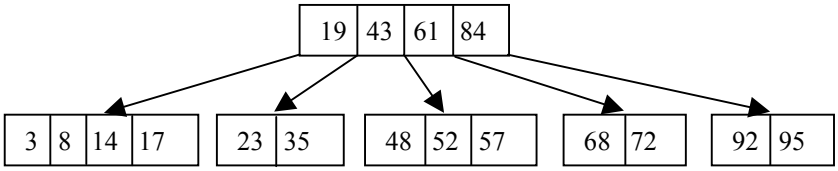
Replace edge (W, X) in the MST above with edge (B, W)

The total weight of the minimum spanning tree is 7 for both MSTs.

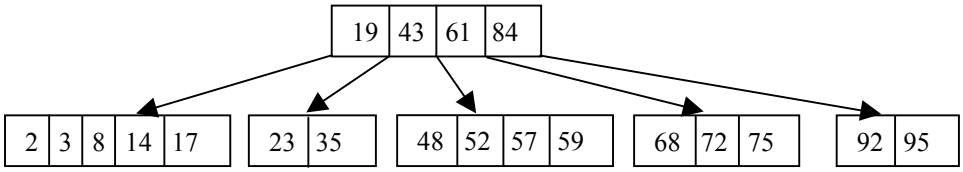
5. [10 points] What are the strongly connected components of G?

There are two strongly connected components: {S, A, B, X, Y} and {C}.

6. Consider a B-tree with t=3. The minimum number of keys in a node will be 2 and the maximum will be 5 (except for the root which may have from 1 to 5 keys). Use the method of splitting shown in class: when a key needs to be inserted into a full node the node is split at the middle key, the middle key is "promoted" to the level above, and then the new key is put into the appropriate node. In a B-tree with t=3, inserting a key into a full node (with keys 0-4) would result in a split such that keys 0-1 are together in a node, keys 3-4 are placed together in a new node, key 2 is "promoted" to the level above, and then the new key (the one being inserted) would be placed in the appropriate leaf node. The starting state for the B-tree is shown below:



a. [10 poitns] Using the starting state shown above, draw a graphical representation of the B-tree after inserting the following keys in the order shown (left to right): 59 75 2



b. [10 points] Using the starting state given in the problem description above, draw a graphical representation of the B-tree after inserting the following keys in the order shown (left to right):
18 11 15