Turn this assignment in electronically using the Curator system. No late assignments will be accepted.

1. [10 points] Simplify the following summations. Assume that *k* is a constant.

a. $2+\sum_{i=1}^{n} 3$

3n+2

b. $\sum_{i=1}^{n} (5n+2)$

$5n^2 + 2n$

c. $\sum_{i=1}^{n} 4i$

$4 * (1/2) * (n^2 + n) = 2(n^2 + n)$

d. $\sum_{i=0}^{n+1} 3$

3(n+2) = 3n + 6

e. $\sum_{i=1}^{n} \left(\sum_{j=i+1}^{n} 8 \right)$

$$8\sum_{i=1}^{n} (n-i) = 4n^2 - 4n$$

f. $\sum_{i=1}^{k} 4n$

4nk

For the next two problems, use the proof format shown below. In the first line, indicate what type of induction you are using (either weak or strong). Note that in order to receive full credit, you must use your inductive assumption when "showing" what you "need to show". You must also show that your base case(s) are true. Be sure to give justifications for all steps.

Proof using (weak | strong) induction:

a) Let S be the set of all integers N > 0 such that:
b) Base case(s):
c) Inductive Assumption:
d) Need to show that:
e) Show:
f) Therefore:

2. [10 points] Use mathematical induction to prove that for all integers N > 0, the integer 2*N is even. For the purposes of this problem, an integer I is even if I mod 2 = 0. You may use the mathematical property that the sum of two even numbers is an even number.

Proof using weak induction:

a) Let S be the set of all integers N > 0 such that:
2*N is even (2*N mod 2 = 0)

b) Base case(s):
N = 1, 2N = 2(1) = 2 which is even because 2 mod 2 = 0 √

c) Inductive Assumption:
Assume that there exists some integer k > 1 such that k is an element of S (i.e. 2k mod 2 = 0).

d) Need to show that:
k + 1 is an element of S (i.e. 2(k+1) mod 2 = 0).

e) Show:
2 * (k + 1) = 2k + 2
2k mod 2 = 0 by Inductive Assumption
2 mod 2 = 0 by algebra / definition of mod
2k + 2 mod 2 = 0 because the sum of two even numbers is even

f) Therefore:
By weak PMI, S contains all integers [1, ∞). Thus, for all integers N > 0, 2N is even.

3. [10 points] Use mathematical induction to prove that for all integers $n > 0$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Proof using weak induction:

a) Let S be the set of all integers N > 0 such that: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

b) Base case(s):

N = 1, $\sum_{i=1}^{1} i = 1 = \frac{1(2)}{2} = 1$ √

N = 2, $\sum_{i=1}^{2} i = 1 + 2 = 3 = \frac{2(3)}{2} = 3$ √

c) Inductive Assumption:
Assume that there exists some integer k > 1 such that k is an element of S. That is, that:

$$\sum_{i=1}^{k} i = 1 + \ldots + k = \frac{k(k+1)}{2}$$

d) Need to show that:

k + 1 is an element of S. That is, that:

$$\sum_{i=1}^{k+1} i = 1 + \ldots + k + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

e) Show:

$$\sum_{i=1}^{k} i = 1 + \ldots + k = \frac{k(k+1)}{2}$$ by Inductive Assumption

$$\sum_{i=1}^{k+1} i = 1 + \ldots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$ by algebra

$$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + \frac{2k+2}{2}$$ by algebra

$$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + \frac{2k+2}{2}$$ by algebra

$$\sum_{i=1}^{k+1} i = \frac{k(k+1)+2k+2}{2}$$ by algebra

$$\sum_{i=1}^{k+1} i = \frac{k^2+k+2k+2}{2}$$ by algebra

$$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$$ by algebra

$$\sum_{i=1}^{k+1} i = \frac{(k+1)((k+1)+1)}{2}$$ by algebra

This last equation is exactly what we needed to show.

f) Therefore:

By weak PMI, S contains all integers [1, ∞). Thus, for all integers N > 0, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$