

**CS 2604**  
**Homework Assignment 3 (Correction 1)**  
**Due: December 5, 2003**

The point value of each problem is shown in [ ]. Each solution must include all calculations and an explanation of why the given solution is correct. In particular, write complete sentences. Your solutions must be prepared electronically and “printed” as a PDF file. The PDF file must be *submitted electronically* to the curator by 4:00 PM on December 5, 2003. **No late homework will be accepted.**

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[10] 1. Assume the following list of keys

T B Z X Q M J

is the initial order of a self-organizing list.

A. For the **count heuristic**, show the lists that occur for this sequence of accesses

B T J M Q M J Z J J J X B J,

and give the individual and total numbers of comparisons performed.

B. For the **move-to-front heuristic**, show the lists that occur, and give the individual and total numbers of comparisons performed, for the same sequence of accesses as in Item A.

C. For the **transpose heuristic**, show the lists that occur, and give the individual and total numbers of comparisons performed, for the same sequence of accesses as in Item A.

D. Suppose you know that the probability of occurrence of each key is given by this table:

KEY	PROBABILITY OF OCCURRENCE
B	$2/7$
J	$3/7$
M	$1/7$
Q	$1/112$
T	$5/112$
X	$1/56$
Z	$1/14$

After a random sequence of 11200 accesses according to the above distribution, what is the expected number of accesses to each key?

- E.** Assume the probability distribution in Item **D**. If you must place the keys in a list with a fixed order (**not** a self-organizing list), what is the optimal<sup>1</sup> order for the list? What is the expected number of comparisons for a single, random access to that list?
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**[10] 2.** ASCII is a 7-bit character code. For purposes of building a hash function, a character  $c$  is often interpreted as a string of 7 bits, as an integer  $i(c)$  between  $0_8$  and  $177_8$ , or as both. See, for example,

<http://www.encypher.net/ascii/>,

where there is an ASCII table from which the bit and integer values are easily derived.

Suppose that keys are two ASCII characters long:  $c_1c_2$ . A possible hash function is

$$h(c_1c_2) = a_1i(c_1) + a_2i(c_2) \pmod{M},$$

where  $a_1$  and  $a_2$  are integers constants and  $M$  is the hash table size. For example, if  $M = 11$ ,  $a_1 = 2$  and  $a_2 = 3$ , then

$$\begin{aligned} h(\text{Aj}) &= 2i(\text{A}) + 3i(\text{j}) \pmod{11} \\ &= 2 \cdot 65 + 3 \cdot 106 \pmod{11} \\ &= 448 \pmod{11} \\ &= 8. \end{aligned}$$

Suppose that  $M = 12$  and that these 8 keys —

**aA    AG    cn    nc    np    mm    nm    mn**

— go into the hash table. Find constants  $a_1$  and  $a_2$  that yield a hash function  $h$  that minimizes the number of collisions of pairs of keys. Justify your claim of having a minimum number of colliding key pairs.

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**[10] 3. Exercise 10.7.** Give a proof by induction on  $k$ .

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**[10] 4. Exercise 11.8.**

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**[10] 5. Exercise 11.21.** Give an example when different trees are obtained.

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<sup>1</sup>**Optimal** means that expected number of comparisons for the list is as small as possible.