## Virginia IIIT Tech

## READ THIS NOW!

- Print your name in the space provided below.
- There are 5 short-answer questions, priced as marked. The maximum score is 100 .
- The grading of each question will take into account whether you obtained a correct solution and how well you presented your analysis and justified your logic. In most cases, as much weight will be given to the presentation and explanation of your analysis as to whether the solution is fully correct. Legibility will be strongly considered in the grading. You may use scratch paper to work out your solution before finalizing it on the exam.
- Externalize! Whether it's a drawing, a table, an equation or something else, externalize! And make the externalization explicit in your answer! Label things for clarity!
- You may use the supplied extra paper for scratch work. Write your name on any scratch work sheets you use and turn those in with your exam.
- All final answers must be written on the test form itself.
- When you have finished, sign the pledge at the bottom of this page and turn in the test.
- This is a closed-book, closed-notes examination.
- No laptops, calculators, cell phones or other electronic devices may be used during this examination.
- Until solutions are posted, you may not discuss this examination with any student who has not taken it.
- Failure to adhere to any of these restrictions is an Honor Code violation.

Name (Last, First) $\qquad$ printed

Pledge: On my honor, I have neither given nor received unauthorized aid on this examination.
signed

1. [20 points] The local chapter of the Association for Computational Excellence held a bake sale to raise money to provide Fortran training to deserving youths. Three roommates, Fernando, Alice, and Hal, shopped at the bake sale. Fernando's purchases included five loaves of pumpkin bread, but he rejected the coconut snoballs. Alice purchased two chocolate chip muffins and one loaf of pumpkin bread. Hal bought ten items, including five chocolate chip muffins. In the end, the roommates had purchased eight of each of the available items.

Exactly what did each roommate purchase at the ACE bake sale?
Note: a good, well-labeled externalization for this problem will serve as an explanation of your logic.

This is easily analyzed by creating a table to organize the given data (shown in blue), which serves as a basis for determining the remaining values (shown in red):

| Person | Pumpkin bread | Coconut snoballs | Choc chip muffins | total units |
| :--- | :---: | :---: | :---: | :---: |
| Fernando | 5 | 0 | 1 | 6 |
| Alice | 1 | 5 | 2 | 8 |
| Hal | 2 | 3 | 5 | 10 |
| total units | 8 | 8 | 8 | 24 |

So:
Fernando bought 5 loaves of pumpkin bread and 1 chocolate chip muffin.
Alice bought 1 loaf of pumpkin bread, 5 coconut snoballs, and 2 chocolate chip muffins.
Hal bought 2 loaves of pumpkin bread, 3 coconut snoballs, and 5 chocolate chip muffins.

## Exposition:

(This is not part of the expected solution, but merely an explanation to supplement the work shown above.)

Aside from Hal's total, the entries in the last column are not strictly necessary, but are useful as a check on the other entries.

My starting point was to fill in that Hal must have bought 2 loaves of pumpkin bread, since a total of 8 loaves were purchased and the given data accounts for the purchases by Fernando and Alice. Then, since Hal purchased a total of 10 items, he must have purchased 3 coconut snoballs.

From there, we immediately see that Alice purchased 5 coconut snoballs.

And, since a total of 8 chocolate chip muffins were bought, Fernando must have purchased the only one that's not yet accounted for.
2. [20 points] Four women own the following musical instruments: Maria, trumpet and bassoon; Harriet, trumpet and flute; Jackie, bassoon and clarinet; Anna, trumpet and oboe. If the oboe is cheaper than the bassoon, the trumpet is more expensive than the flute, the trumpet is cheaper than the oboe, and the flute is more expensive than the clarinet, who owns the most expensive pair of instruments?

Note: a good, well-labeled externalization for this problem will serve as an explanation of your logic.
For this we must determine a linear ordering of the instruments, based upon what we are given about their relative prices.

We know that the oboe is cheaper than the bassoon, the trumpet is cheaper than the oboe, the flute is cheaper than the trumpet, and that the clarinet is cheaper than the flute.

Hence we have the following linearization:
clarinet flute trumpet oboe bassoon

It is helpful to form a table so we can mark off the possession information:

|  | clarinet | flute | trumpet | oboe | bassoon |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Maria |  |  | $X$ |  | $X$ |
| Harriet |  | $X$ | $X$ |  |  |
| Jackie | $X$ |  |  |  | $X$ |
| Anna |  |  | $X$ | $X$ |  |

So, Maria owns the most expensive pair of instruments.

## Exposition:

(This is not part of the expected solution, but merely an explanation to supplement the work shown above.)

Now, Maria's two instruments, trumpet and bassoon, must be more expensive than Harriet's two, flute and trumpet, since the bassoon is more expensive than the flute.

And, Maria's two instruments must be more expensive than Jackie's two, bassoon and clarinet, since the trumpet is more expensive than the clarinet.

And, finally, Maria's two instruments must be more expensive than Anna's two, since the bassoon is more expensive than the oboe.
3. [20 points] In the town of Gridburg, all streets are straight and run either north-south or east-west. Streets with names that begin with a vowel and end with a consonant always run north-south. And, streets with names that begin with a consonant and end with a vowel always run east-west. Streets whose names do not follow either of those patterns may run either way.

Center Street intersects Agnes Avenue. Is it possible or impossible that Center Street also intersects Parabola Parkway? If that is possible, can you determine whether it must be the case? If it is impossible that Center Street intersects Parabola Parkway, can you determine whether or not Agnes Avenue and Parabola Parkway do intersect?

Note: a good, well-labeled externalization for this problem, with a little explanation, will serve as an explanation of your logic.

There is an ambiguity in the question. For Center Street, does "name" refer to "Center" or to "Center Street"? Either interpretation is acceptable as long as the answer makes clear you used a consistent approach.

Taking the first interpretation for "name", we see that Agnes Avenue must run north-south, and therefore that Center Street must run east-west, since it intersects Agnes Avenue. And, Parabola Parkway must also run east-west:


So, Center Street and Parabola Parkway are parallel and they cannot possibly intersect, UNLESS they meet head-on as shown above.

On the other hand, it is possible, but not necessary, that Agnes Avenue and Parabola Parkway intersect:


Now, taking the second interpretation for "name", we have a difficulty. Agnes Avenue begins and ends with a vowel, so we cannot say whether it runs east-west or north-south; the same logic holds for Center Street and Parabola Parkway. So, using this interpretation, it is possible that any pair of streets do intersect, but we cannot guarantee than any pair does intersect:

4. [20 points] Suppose that you are participating in a group that is attempting to solve the problem given below. The group is following the solver/listener approach discussed in class, and you are playing the role of listener.

A certain mechanical clock runs slow; it indicates that one hour has passed when actually 62 minutes have passed. If the clock is set correctly at $1: 00 \mathrm{pm}$, what is the correct time when the clock next indicates the time is 6:30 pm?

After reading the question aloud, the solver continues by offering the following comments:
The clock shows the wrong time... if it's right at 1:00 pm then at 2:00 pm the clock will show an earlier time... it will show it's $1: 58 \mathrm{pm}$.

Hmm... this looks messy.

Give an example of something useful you could say at this point that would stay within your assigned role. Explain why your answer is appropriate.

The solver has made a strange assertion. Since the clock runs slowly, when the time is actually 2:00 pm, the clock will show an earlier time. However, the problem statement implies that when the clock shows 2:00 pm the actual time will be 2:02 pm. That is NOT the same as saying that when the actual time is $2: 00 \mathrm{pm}$ the clock will show $1: 58 \mathrm{pm}$. If the clock is "losing time" at a uniform rate, it will actually have "lost" slightly less than 2 minutes when the actual time is 2:00 pm...

But as a listener, I cannot simply say "You are wrong, here's the right inference."

Here are a few potentially useful questions I could ask:

- Does the clock actually lose 2 minutes as 60 minutes pass?
- Does the clock lose time steadily? Or in spurts? Does that matter?
- What time would it actually be when the clock showed 2:00 pm?

5. [20 points] Analyze the following premises:
A) Nat E. Newcomb owns two suits, one blue and one brown.
B) Whenever he wears his blue suit and a blue shirt, he also wears a blue tie.
C) He always wears either a blue suit or white socks.
D) He never wears the blue suit unless he is also wearing either a blue shirt or white socks.
E) Whenever he wears white socks, he also wears a blue shirt.
F) Today, Nat is wearing a gold tie.

What else must he be wearing today? What else could he be wearing today?

## Note: For this problem, explain all your inferences carefully. Be precise and complete.

From premise F, we know Nat is not wearing his blue tie today.

From that fact, and premise B, we know that it's not true that "he wears his blue suit and a blue shirt" today. So, we know he's not wearing his blue suit or he's not wearing his blue shirt, or he's not wearing either.

Could he be wearing his blue suit? If so, then by $D$, he must also be wearing his blue shirt or white socks, but not both ("either... or"). But, he can't be wearing both the blue suit and the blue shirt, from our earlier inference. So, if he wears his blue suit, then he must not be wearing the blue shirt, and he must be wearing white socks.

But... from $C$, if he's wearing his blue suit then he cannot be wearing white socks. That's a contradiction.

So, no, he cannot be wearing his blue suit, and so by $C$ he must be wearing white socks.

Could he also not be wearing his blue shirt? If not, then from $E$ he cannot be wearing white socks. But then, from $C$ he must be wearing his blue suit, which has already been ruled out.

So, he must be wearing his blue shirt.

Nothing else can be affirmed. He may be wearing his brown suit, or not.

So, to sum up... he must be wearing the gold tie, and his blue shirt, and white socks.

He cannot be wearing his blue suit, but he could be wearing his brown suit.

