CS 1704

Introduction to Data Structures and Software Engineering

Sorting Terms & Definitions

- Internal sorts holds all data in RAM
- External sorts use Files
- Ascending Order:
  - Low to High
- Descending Order:
  - High to Low
- Stable Sort
  - Maintains the relative order of equal elements, *in situ*.
  - Desirable if list is almost sorted or if items with equal values are to also be ordered on a secondary field.

Comparing Sorting Algorithms

- Program efficiency
  - Overall program efficiency may depend entirely upon sorting algorithm => clarity must be sacrificed for speed.
- Sorting Algorithm Analysis
  - Performed upon the "overriding" operation in the algorithm:
    - Comparisons
    - Swaps

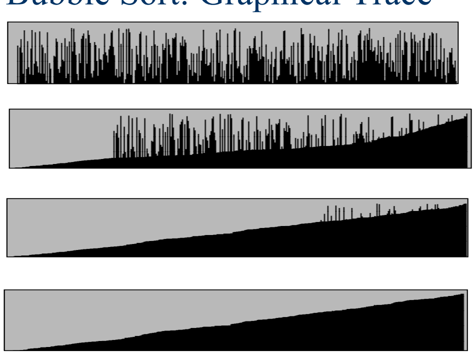
```
void swap( int& x, int& y) {
   int tmp = x;
   x = y;
   y = tmp;
}
```

Bubble Sort

- *Bubble* elements down (up) to their location in the sorted order.

```
for (i = 0; i < n-1; i++)
    for (j = n-1; j > i; j--)
        if (A[j] < A[j-1])
            swap(A[j],A[j-1]);
```

Bubble Sort: Graphical Trace



Bubble Sort: Analysis

- if-statement:
  - 1 compare and in worst case, 1 swap
- inner for-loop:
  - body executed for j-values from n-1 down to i+1, or n-i-1 times
  - each execution of body involves 1 compare and up to 1 swap
- outer for-loop:
  - body executed for i-values from 0 up to n-2 (or 1 to n-1)
  - each execution of body involves n-i-1 compares and up to n-i-1 swaps

Bubble Sort: Analysis

- So in the worst case, the number of swaps equals the number of compares, and is:

$$\sum_{i=1}^{n-1}(n-i-1) = n(n-1) - \frac{1}{2}(n-1)(n-2) - (n-1)$$

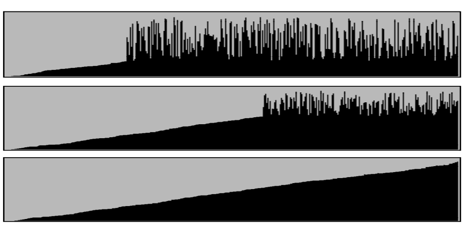
- Which is clearly $O(n^2)$.

Selection Sort

- In the i^{th} pass, select the element with the lowest value among A[i], ..., A[n-1], & swap it with A[i].
- Results after i passes: the i lowest elements will occupy A[0], ..., A[i] in sorted order.

```
for (Begin = 0; Begin < Size - 1; Begin++) {
   SmallSoFar = Begin;
   for (Check = Begin + 1; Check < Size; Check++) {
      if (aList[Check] < aList[SmallSoFar])
         SmallSoFar = Check;
   }
   swap(aList[Begin], aList[SmallSoFar]);
}
```

Selection Sort: Graphical Trace



Selection Sort: Analysis

- if-statement: 1 compare
- inner for-loop:
  - body executed n-i-1 times (i is Begin and n is Size)
  - each execution of body involves 1 compare and no swaps
- outer for-loop:
  - body executed n-1 times
  - each execution of body involves n-i-1 compares and 1 swap

Selection Sort: Analysis

- So in the worst case, the number of swaps is n – 1, and the number of compares is:

$$\sum_{i=1}^{n-1}(n-i-1) = n(n-1) - \frac{1}{2}(n-1)(n-2) - (n-1)$$

- which is clearly O(n2) and the same as for BubbleSort.

Duplex Selection Sort

- Min / Max Sorting
  - algorithm passes thru the array locating the min and max elements in the array A[i], ..., A[n-i+1]. Swapping the min with A[i] and the max with A[n-i+1].
  - Results after the ith pass: the elements A[1], ..., A[i] and A[n-i+1], ..., A[n] are in sorted order.
  - What would be the Big Oh?

Duplex Selection Sort Analysis

- Without going through the figures,
  - Duplex Selection Sort is another O(N^2) sort algorithm.
  - But, the coefficient IS better than for BubbleSort!

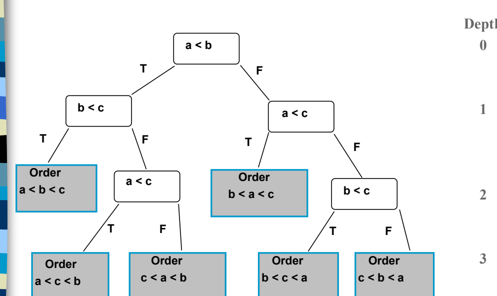
Sorting Thoughts

- **Comparison-Based Sorting**
  - **Algorithms which compare element values to each other**
  - **What is the minimum number of comparisons, required to sort N elements using a comparison-based sort?**
- **Is a Queue a type of sorting?**

Comparison Tree

- **Binary** tree (hierarchical graph ≤ 2 branches per node) which contains comparisons between 2 elements at each non-leaf node & containing element orderings at its leaf (terminal) nodes.

Comparison Tree for 3 Elements



Comparison Tree Continued

- Any of the 3 elements (a, b, c) could be first in the final order. Thus there are 3 distinct ways the final sorted order could start.
- After choosing the first element, there are two possible selections for the next sorted element.
- After choosing the first two elements there is only 1 remaining selection for the last element.
- Therefore selecting the first element one of 3 ways, the second element one of 2 ways and the last element 1 way, there are 6 possible final sorted orderings = **3 * 2 * 1 = 3!**

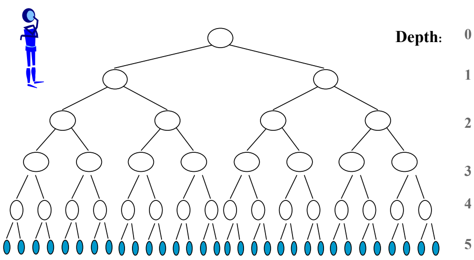
Order Tree for sorting N

- Any of the N elements (1 ... N) could be first in the final order. Thus there are N distinct ways the final sorted order could start.
- After choosing the first element, there are N-1 possible selections for the next sorted element.
- After choosing the first two elements there N-2 possible selections for the next sorted element, etc.
- Therefore selecting the first element one of N ways, the second element one of N-1 ways, etc., there are N * (N-1) * (N-2) * . . . * 2 * 1 possible final sorted orderings which = **N!**

Comparisons for Sorting N

- The comparison tree for N elements must have **N!** leaf nodes. Each leaf node contains one of the possible orderings of all of the N elements.
- Consider the previous comparison tree for 3 elements, all of the leaf nodes are at a **depth** of either 2 or **3** > $\lfloor \log_2 \mathbf{3!} \rfloor$
- The comparison tree for 4 elements must contain **4!**=24 leaf nodes, all of which would be at a depth of either 4 or **5** > $\lfloor \log_2 \mathbf{4!} \rfloor$
- The "floor" $\lfloor\ \rfloor$ symbol means the largest whole number that is less than the number

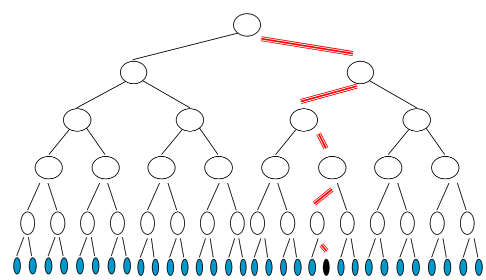
Depth vs. N



General Comparison Trees

- The comparison tree for N elements must contain **N!** leaf nodes, all of which would be at a depth > $\lfloor \log_2 \mathbf{N!} \rfloor$
- The minimal number of comparisons required to sort a specific (unsorted) ordering is equal to the depth from the root to a leaf.

Minimal # of Comparisons



N log N

- Since the depth of all leaf nodes is > $\lfloor \log_2 \mathbf{N!} \rfloor$ in a comparison tree, the minimal number of comparisons to sort a specific initial ordering of N elements is > $\lfloor \log_2 \mathbf{N!} \rfloor$
- Stirling's Approximation for $\log_2(\mathbf{N!})$ can be used to determine a lower bound for $\log_2(\mathbf{N!})$ which is **O(N log N)**
- No comparison based sorting algorithm can sort faster than
  - **O(N log N)**

Quick Sort

- Select an item in the array as the pivot key.
- Divide the array into two partitions: a left partition containing elements < the pivot key and a right partition containing elements > the pivot key.

Quicksort Trace

Start with i and j pointing to the first & last elements, respectively.

```
Select the pivot (3):        [3  1  4  1  5  9  2  6  5  8]
                              L                          R
Swap the end elements, then move L, R inwards.
                             [8  1  4  1  5  9  2  6  5  3]
                              L                    R
Swap, and repeat:            [2  1  4  1  5  9  8  6  5  3]
                                    L  R
Swap, and repeat:            [2  1  1 | 4  5  9  8  6  5  3]
                                    R    L
```

Quicksort

```
int Partition(Item A[], int start, int end,
                                  const Item& pivot ){
   int L = start, R = end;

   do {
       swap( A[L] , A[R] );
       while (A[L] < pivot )      L++;
       while (!(A[R] < pivot))    R--; //< overloaded
   } while (R > L);
   return (L);
}
```

Pivoting

- Partitioning test **requires** at least 1 key with a value < that of the pivot, and 1 value $\geq$ to that of the pivot, to execute correctly.
- Therefore, pick the greater of the first two distinct values (if any).
  - OR Try and pick a pivot such that the list is split into equal size sublists, (a speedup that should cut the number of partition steps to about 2/3 that of picking the first element for the pivot).
    - Choose the middle (median) of the first 3 elements.
    - Pick k elements at random from the list, sort them & use the median.

Find Pivot

```
const int  MISSING = -1;
int FindPivot(const Item A[], int start, int end ) {
   Item firstkey;  //value of first key found
   int pivot;      //pivot index
   int k;          //run right looking for other key

   firstkey = A[start];
   //return -1 if different keys are not found
   pivot = MISSING;
   k = start + 1;
   //scan for different key
   while ( (k <= end) && (pivot == MISSING) )
      if (firstkey < A[k])        //select key
         pivot = k;
      else if (A[k] < firstkey)
         pivot = start;
      else
         k++;
   return pivot;
}
```

Quick Sort Function

```
const int  MISSING = -1;
void QuickSort( Item A[], int start, int end ) {
   // sort the array from start ... end
   Item pivotKey;
   int  pivotIndex;
   int  k;         //index of partition >= pivot

   pivotIndex = FindPivot( A, start, end );
   if (pivotIndex != MISSING) {
      pivotKey = A[pivotIndex];
      k = Partition( A, start, end, pivotKey );
      QuickSort( A, start, k-1 );
      QuickSort( A, k, end );
   }
}
```

Average Case

- quicksort is based upon the intuition that swaps, (moves), should be performed over large distances to be most effective.
- quicksort's average running time is faster than any currently known $O(n \log_2 n)$ internal sorting algorithms (by a constant factor).
- For very small n (e.g., $n \leq 16$) a simple $O(n^2)$ algorithm is actually faster than Quicksort.
  - Optimization: When the sublist is small, use another sorting algorithm, (selection).

Worst Case

- In the worst case, every partition might split the list of size j - i + 1 into a list with 1 element, and a list with j - i elements.
- A partition is split into sublists of size 1 & j-i when one of the first two items in the sublist is the largest item in the sublist which is chosen by findpivot.
- When will this worst case partitioning always occur?
- O(N^2)

Iterative Version is Posted

- Iterative implementation requires using a stack to store the partition bounds remaining to be sorted.

```
struct StackItem {
    int low, hi;
};
```

- At the end of any given partition, only one subpartition need be stacked.

Other Quicksort Optimizations

- All function calls should be replaced by inline code to avoid function overhead.
- Current partition bounds should be held in register variables.
- With large data records, swap pointers instead of copying records
  - We're accepting the cost of additional pointer dereferences to avoid the cost of some data copying.
- Carefully investigate the average data arrangement in order to select the optimal sorting algorithm.
  - For example, to identify special cases within Quicksort

Special Case: Mapping

- Better than N log N
  - If sort key (member) consists of consecutive (unique) N integers they can be easily mapped onto the range 0 .. N-1 & sorted.
  - If the N elements are initially in array A, then:

```
Item A[], B[];

for (int i = 0; i < N; i++ )
     B[A[i].GetKey() % N] = A[i];
```

Mapping

- takes O(n) time.
- Requires exactly 1 record with each key value!
- Of course, this is a very special circumstance…
- Special case of Bin Sorting. (If integers are not consecutive, but within a reasonable range, bit flags can be used to denote empty array slots.)

Bin Sorting

- Assume we need to sort an array of integers in the range 0-99:
- Assume we have an array of 10 linked lists (bins) for storage.
- First make a pass through the list of integers, and place each into the bin that matches its 1's digit.
- Then, make a second pass, taking each bin in order, and place each integer into the bin that matches its 2's digit, etc.

Bin Sorting

- Now if you just read the bins, in order, the elements will appear in ascending order.
  – Assuming no Bin with > 1 element
  – Otherwise, use another sort technique to sort bins
- Each pass takes O(N) work, and the number of passes is just the number of digits in the largest integer in the original list.
- That beats QuickSort, but only in a somewhat special case. (When each bin has 1 element)
- Binsort Implementation will be posted online.