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The following formulas may be easily proved using Mathematical Induction (Math 2534).

Let  $N \geq 0$ , let  $A$ ,  $B$ , and  $C$  be constants, and let  $f$  and  $g$  be any functions. Then:

$$\sum_{k=1}^N Cf(k) = C \sum_{k=1}^N f(k)$$

**S1: factor out constant**

$$\sum_{k=1}^N (f(k) \pm g(k)) = \sum_{k=1}^N f(k) \pm \sum_{k=1}^N g(k)$$

**S2: separate summed terms**

$$\sum_{k=1}^N C = NC$$

**S3: sum of constant**

$$\sum_{k=1}^N k = \frac{N(N+1)}{2}$$

**S4: sum of  $k$**

$$\sum_{k=1}^N k^2 = \frac{N(N+1)(2N+1)}{6}$$

**S5: sum of  $k$  squared**

Let  $b > 0$ , and let  $x > 0$ . Then the logarithm of x to base b is the power to which b must be raised in order to get x. That is:

$$y = \log_b(x) \quad \text{if, and only if,} \quad x = b^y$$

Let  $A > 0$  and  $B > 0$  and  $N$  be an integer. If no base is shown, then the rule holds for any base.

$$\log(AB) = \log(A) + \log(B)$$

**L1: log of a product**

$$\log\left(\frac{A}{B}\right) = \log(A) - \log(B)$$

**L2: log of a quotient**

$$\log(A^N) = N \log(A)$$

**L3: log of a power**

$$\log(1) = 0$$

**L4: log of 1**

$$\log_b(b^N) = N$$

**L5: log of power of base**

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)} \quad \text{for any } b > 0 \text{ and } a > 0$$

**L6: change of base**