

Instructions: This homework assignment focuses on algorithm analysis. Submit your answers via the Curator as Quiz: Analysis.

For questions 1 through 5, apply the rules given in the course notes and in class to determine the best big-O classification of the given function. All logarithms are base 2.

1. $f(n) = 4n^2 + 97n \cdot \log n + 1000$ is $O(\underline{\hspace{2cm}})$
- | | | |
|----------------|--------------------|------------------|
| 1) $O(1)$ | 4) $O(n \log n)$ | 7) $O(2^n)$ |
| 2) $O(\log n)$ | 5) $O(n^2)$ | 8) None of these |
| 3) $O(n)$ | 6) $O(n^2 \log n)$ | |
2. $f(n) = 17n + 43n \cdot \log n + \log n$ is $O(\underline{\hspace{2cm}})$
- | | | |
|----------------|--------------------|------------------|
| 1) $O(1)$ | 4) $O(n \log n)$ | 7) $O(2^n)$ |
| 2) $O(\log n)$ | 5) $O(n^2)$ | 8) None of these |
| 3) $O(n)$ | 6) $O(n^2 \log n)$ | |
3. $f(n) = 3n + 100,000$ is $O(\underline{\hspace{2cm}})$
- | | | |
|----------------|--------------------|------------------|
| 1) $O(1)$ | 4) $O(n \log n)$ | 7) $O(2^n)$ |
| 2) $O(\log n)$ | 5) $O(n^2)$ | 8) None of these |
| 3) $O(n)$ | 6) $O(n^2 \log n)$ | |
4. $f(n) = 3n(\log n + 7n)$ is $O(\underline{\hspace{2cm}})$
- | | | |
|----------------|--------------------|------------------|
| 1) $O(1)$ | 4) $O(n \log n)$ | 7) $O(2^n)$ |
| 2) $O(\log n)$ | 5) $O(n^2)$ | 8) None of these |
| 3) $O(n)$ | 6) $O(n^2 \log n)$ | |
5. $f(n) = 1000n^2 + 0.001 \cdot 2^n$ is $O(\underline{\hspace{2cm}})$
- | | | |
|----------------|--------------------|------------------|
| 1) $O(1)$ | 4) $O(n \log n)$ | 7) $O(2^n)$ |
| 2) $O(\log n)$ | 5) $O(n^2)$ | 8) None of these |
| 3) $O(n)$ | 6) $O(n^2 \log n)$ | |

6. Using the rules given in the notes and class, Analyze the following code fragment to estimate $T(N)$.

```
int i, Sum;
i = N;
Sum = 0;
while (i > 0) {
    for (int j = 0; j < i; j++)
        Sum++;
    cout << Sum << end;
    i--;
}
```

Based on your analysis, what is the best big-O classification for $T(N)$?

- | | | |
|----------------|------------------|------------------|
| 1) $O(1)$ | 3) $O(N)$ | 5) $O(N^2)$ |
| 2) $O(\log N)$ | 4) $O(N \log N)$ | 6) None of these |

7. Using the rules given in the notes and class, Analyze the following code fragment to estimate $T(N)$.

```
int Sum;
Sum = 0;
for (int i = 1; i <= N; i++) {           //Hint: you may assume N is a power of 2
    for (int j = 1; j <= N; j = 2*j)
        Sum++;
}
```

Based on your analysis, what is the best big-O classification for $T(N)$?

- | | | |
|----------------|------------------|------------------|
| 1) $O(1)$ | 3) $O(N)$ | 5) $O(N^2)$ |
| 2) $O(\log N)$ | 4) $O(N \log N)$ | 6) None of these |
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For questions 8 through 10, consider the problem of performing a search on a sorted linked list containing N nodes.

8. In the best case, searching a sorted linked list containing N nodes would be $O(\underline{\quad})$ for comparisons.

- | | | |
|----------------|------------------|------------------|
| 1) $O(1)$ | 3) $O(N)$ | 5) $O(N^2)$ |
| 2) $O(\log N)$ | 4) $O(N \log N)$ | 6) None of these |

9. In the worst case, searching a sorted linked list containing N nodes would be $O(\underline{\quad})$ for comparisons.

- | | | |
|----------------|------------------|------------------|
| 1) $O(1)$ | 3) $O(N)$ | 5) $O(N^2)$ |
| 2) $O(\log N)$ | 4) $O(N \log N)$ | 6) None of these |

10. In the average case, assuming that each element of the list is equally likely to be targeted, searching a sorted linked list containing N nodes would be $O(\underline{\quad})$ for comparisons.

- | | | |
|----------------|------------------|------------------|
| 1) $O(1)$ | 3) $O(N)$ | 5) $O(N^2)$ |
| 2) $O(\log N)$ | 4) $O(N \log N)$ | 6) None of these |
-

11. Suppose that the execution of a particular algorithm requires carrying out $T(N)$ operations, where N is the number of inputs that must be processed and

$$T(N) = 2N^2 + 7N/2 + 42$$

Assume the algorithm will be executed on hardware capable of performing 10^6 operations per second. What is the largest size problem (value of N) for which the algorithm can be carried out in 60 seconds? (Pick the closest answer.)

- | | | |
|-----------|-----------|-------------|
| 1) 1 | 5) 20,000 | 9) 50,000 |
| 2) 5000 | 6) 25,000 | 10) 100,000 |
| 3) 10,000 | 7) 30,000 | |
| 4) 15,000 | 8) 40,000 | |
-

12. Suppose that the execution of a particular algorithm requires carrying out $T(N)$ operations, where N is the number of inputs that must be processed and

$$T(N) = N \log N + 7N + 42$$

Assume the algorithm will be executed on an input of size 2^{20} , using the same hardware described in question 11. How long will it take (in seconds)? (Pick the closest answer.)

- | | | |
|-------|--------|-----------|
| 1) 1 | 5) 40 | 9) 1000 |
| 2) 10 | 6) 50 | 10) 10000 |
| 3) 20 | 7) 60 | |
| 4) 30 | 8) 120 | |

13. Repeat question 12, assuming the input is twice as large.

- | | | |
|-------|--------|-----------|
| 1) 1 | 5) 40 | 9) 1000 |
| 2) 10 | 6) 50 | 10) 10000 |
| 3) 20 | 7) 60 | |
| 4) 30 | 8) 120 | |

14. Suppose that the execution of a particular algorithm requires carrying out $T(N)$ operations, where N is the number of inputs that must be processed and

$$T(N) = N^2 + 2N + 1$$

Assume the algorithm will be executed on an input of size 2^{20} , using the same hardware described in question 11. How long will it take? (Pick the closest answer.)

- | | | |
|---------------|-------------|--------------|
| 1) 10 minutes | 5) 12 hours | 9) 50 days |
| 2) 20 minutes | 6) 1 day | 10) 100 days |
| 3) 60 minutes | 7) 10 days | |
| 4) 6 hours | 8) 20 days | |

15. Repeat question 14, assuming the input is twice as large.

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|---------------|-------------|--------------|
| 1) 10 minutes | 5) 12 hours | 9) 50 days |
| 2) 20 minutes | 6) 1 day | 10) 100 days |
| 3) 60 minutes | 7) 10 days | |
| 4) 6 hours | 8) 20 days | |