Facts from Mathematics

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Summation Formulas

The following formulas may be easily proved using Mathematical Induction (Math 2534).

Let N ≥ 0, let A, B, and C be constants, and let *f* and *g* be any functions. Then:

$$\boxed{\sum_{k=1}^{N} Cf(k) = C\sum_{k=1}^{N} f(k)}$$

S1: factor out constant

$$\boxed{\sum_{k=1}^{N} (f(k) \pm g(k)) = \sum_{k=1}^{N} f(k) \pm \sum_{k=1}^{N} g(k)}$$

S2: separate summed terms

$$\boxed{\sum_{k=1}^{N} C = NC}$$

S3: sum of constant

$$\boxed{\sum_{k=1}^{N} k = \frac{N(N+1)}{2}}$$

S4: sum of *k*

$$\boxed{\sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}}$$

S5: sum of *k* squared

Laws of Logarithms

Let b > 0, and let x > 0. Then the logarithm of x to base b is the power to which b must be raised in order to get x. That is:

$$y = \log_b(x) \quad \text{if, and only if,} \quad x = b^y$$

Let A > 0 and B > 0 and N be an integer. If no base is shown, then the rule holds for any base.

$$\boxed{\log(AB) = \log(A) + \log(B)}$$

L1: log of a product

$$\boxed{\log\left(\frac{A}{B}\right) = \log(A) - \log(B)}$$

L2: log of a quotient

$$\boxed{\log\left(A^N\right) = N\log(A)}$$

L3: log of a power

$$\boxed{\log(1) = 0}$$

L4: log of 1

$$\boxed{\log_b\left(b^N\right) = N}$$

L5: log of power of base

$$\boxed{\log_b(x) = \frac{\log_a(x)}{\log_a(b)}}$$ **for any b > 0 and a > 0**

L6: change of base