Cortical High-Density Counterstream Architectures
It’s not a small world after all

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Tuesday, August 2, 2016
Graph Density

Q — What is the density of a graph?
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Density is the ratio of edges present to the total number of edges that could possibly be present in a graph.

— A
Density Examples

Potential Connections:
\[ PC = \frac{n \times (n-1)}{2} \]

Network Density:
\[ \frac{Actual \ Connections}{Potential \ Connections} \]

Examples:

A
Nodes (n): 2
Potential Connections: 1  \( (2 \times 1)/2 \)
Actual Connections: 1
Network Density: 100%  \( (1/1) \)

B
Nodes (n): 3
Potential Connections: 3  \( (3 \times 2)/2 \)
Actual Connections: 3
Network Density: 100%  \( (3/3) \)

C
Nodes (n): 3
Potential Connections: 3  \( (3 \times 2)/2 \)
Actual Connections: 2
Network Density: 66.7%  \( (2/3) \)
Figure 1A: Comparison with Past Results

![Graph showing average path length vs. graph density with data points for different studies.](image-url)
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- Who produced the data set and wrote the cited studies (References 8,11) challenging the small world hypothesis? **The authors of this paper!**
The subgraph is described as 'edge-complete'. What does this mean?
Edge-completeness

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- What is ’edge-incompleteness’? See the bottom of the first page. **Pathways are untested in edge-incomplete subgraphs? The definition is unclear. Are these definitions consistent?**
Why is this graph more dense than prior works?
NFPs: Newfound Projections

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- What is the technique that the authors used to obtain their data? Retrograde tracers.
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- Weakened viral strains are injected and carried by retrograde transport back to the source of a neural connection.
- Why can viruses be carried by retrograde transport? They have evolved to exploit this natural cellular mechanism!
Axonal Transport: Where do we inject tracers?
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- Inject at the synapse, on the right. This is the destination of a neural connection.
Axonal Transport: Where do we inject tracers?

- What is the FLN?
What is the FLN? **Fraction of labeled neurons.**

If we injected tracer in area $i$ and were targeting the cell bodies in area $j$, the FLN is the ratio:

$$f_{ij} = \frac{\text{Number of labeled neurons in area } j}{\text{Total number of extrinsic (not in } i) \text{ labeled neurons}}$$
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\]

- We use FLN as a weight for our graph
What do these new edges imply about SW?
Small World Refresher

The diagram shows the ratio of the clustering coefficient $C(p) / C(0)$ and the average path length $L(p) / L(0)$ for varying values of $p$. The points indicate the measured values, while the lines are a visual aid to connect the data points.
Figure 1B: The small world property is density-dependent
Figure 1B Data

Where does this data come from?
Figure 1B Data

- Hypothetical 1000 node ring lattices.
- The path length and clustering are displayed over a range of rewiring probabilities.
- Path length and clustering values are listed for densities from 6% to 66% and color-coded.
At what density does the SW property stop occurring?
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What do the authors conclude about the cortex?
Figure 1B Data

- At what density does the SW property stop occurring? 42%
- What do the authors conclude about the cortex? It’s not such a small world after all!
Thresholding

- Thresholding impacts the density. Why do the authors not use thresholding?
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- Thresholding impacts the density. Why do the authors not use thresholding? **Eliminating pathways with fewer than 10 neurons per tracer drops the density from 66% to 53%**
- 37% of the pathways that would be eliminated have been reported in previous publications.
- Larger injections may result in stronger connections in those same pathways.
Vertex Cover Problem

- Given an undirected graph $G(V, E)$, a subset $S \subseteq V$ is a vertex cover if every edge in $E$ is incident to at least one vertex in $S$.
- Can you find a vertex cover in this image?
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- We are not interested in vertex cover explicitly, but in a different problem called 'dominating set analysis' that is similar.
Dominating Set Analysis

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How is this similar to the vertex cover problem? What does that tell us about the complexity of computing a dominating set?
Minimum Fully Dominating Set (MDS)

- The minimum fully dominating set (MDS) is the smallest set of nodes that forms a dominating set.
Figure 2A: Dominating Sets
The size of the data set makes it easy to find dominating sets visually.
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What is the size of the MDS?
Figure 2A: Dominating Sets

- The size of the data set makes it easy to find dominating sets visually.
- What is the size of the MDS?
- Does the MDS increase if we take away the NFPs (the red boxes)?
Impact of NFPs on Dominating Sets

A

Percent of possible dominating sets

Percent of dominated areas

B

Percent of possible dominating sets

Percent of dominated areas
Aside: Size of the Data Set

Those of you who downloaded the data set, how many monkeys were there?
Aside: Size of the Data Set

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- Those of you who downloaded the data set, how many monkeys were there? 39.
- Is this a large data set for a biology project? How expensive and difficult is it to take care of 39 monkeys?
Aside: Size of the Data Set

- Those of you who downloaded the data set, how many monkeys were there? **39.**
- Is this a large data set for a biology project? How expensive and difficult is it to take care of 39 monkeys?
- Is this a large data set in terms of graph theory? Our subgraph contains 29 nodes. What does this tell us about the authors’ conclusions?
Conclusions so far

- The macaque cortex is not small world because the small world property disappears when density exceeds 42%.
- The cortex is more dense than previously reported.
- Newfound projections connect dissimilar areas across long distances with a highly specific purpose.