Perceptron

Machine Learning
CS5824/ECE5424
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Virginia Tech

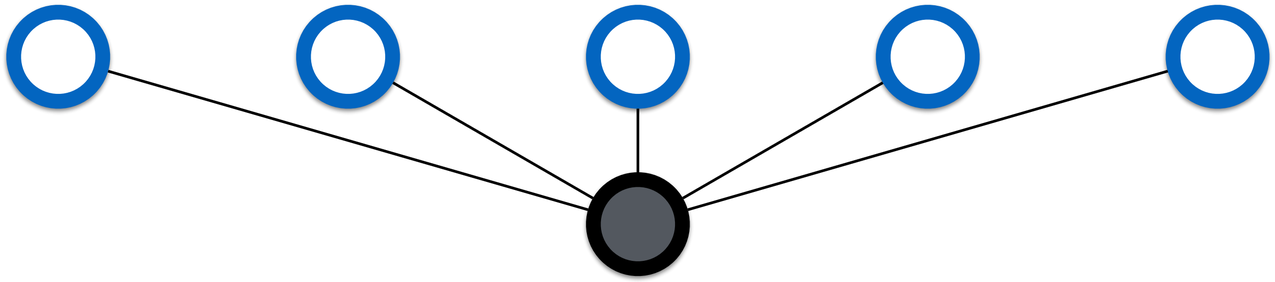
Plan

- Perceptron
- Multi-class perceptron

Perceptron

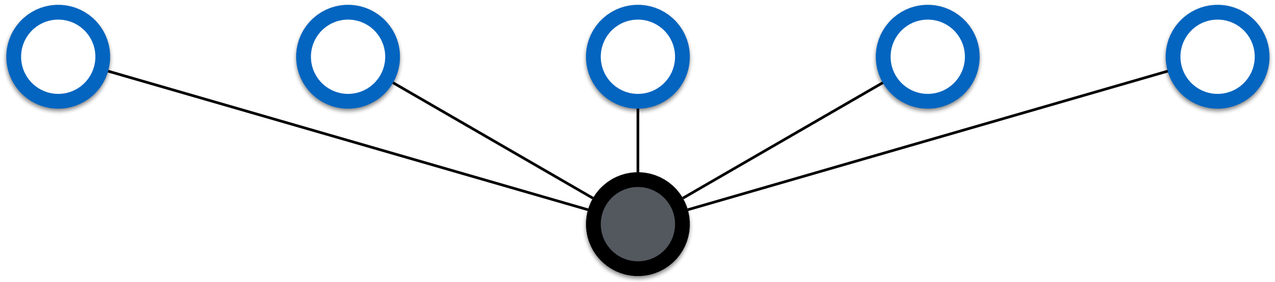
Perceptron

- AKA single-layer neural network



Perceptron

- AKA single-layer neural network

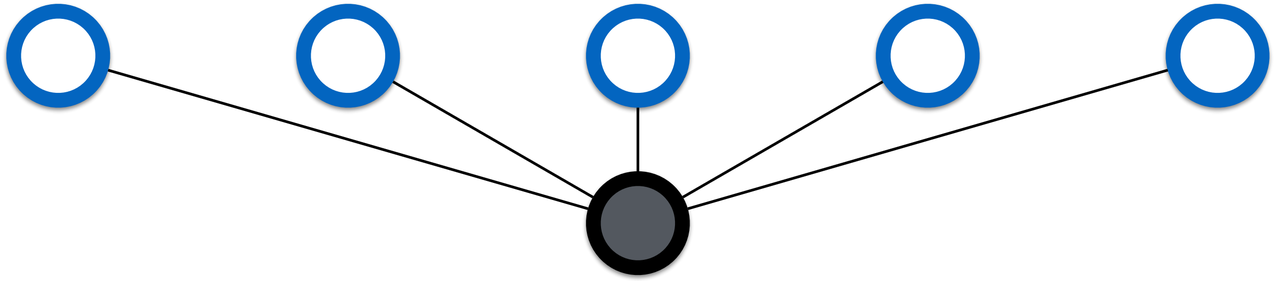


- linear classifier

$$f(x) = \text{sign}\left(\sum_i w_i x_i\right)$$

Perceptron

- AKA single-layer neural network

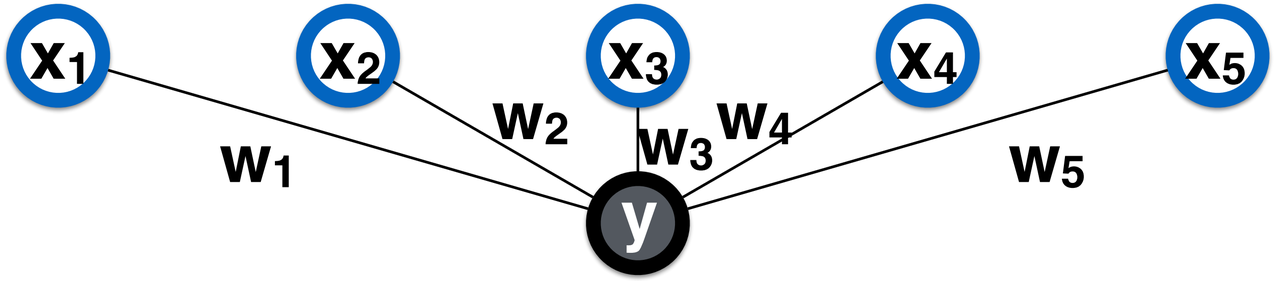


- linear classifier

$$f(x) = \text{sign}\left(\sum_i w_i x_i\right) = \begin{cases} +1 & \text{if } \sum_i w_i x_i \geq 0 \\ -1 & \text{if } \sum_i w_i x_i < 0 \end{cases}$$

Perceptron

- AKA single-layer neural network

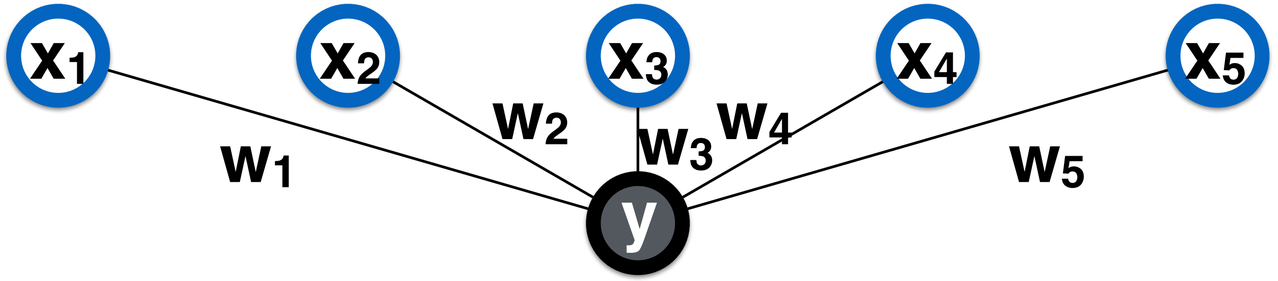


- linear classifier

$$f(x) = \text{sign}\left(\sum_i w_i x_i\right) = \begin{cases} +1 & \text{if } \sum_i w_i x_i \geq 0 \\ -1 & \text{if } \sum_i w_i x_i < 0 \end{cases}$$

Perceptron

- AKA single-layer neural network



- linear classifier

$$f(x) = \text{sign}\left(\sum_i w_i x_i\right) = \begin{cases} +1 & \text{if } \sum_i w_i x_i \geq 0 \\ -1 & \text{if } \sum_i w_i x_i < 0 \end{cases}$$

- online learner

Online Training

- Get example **x** and oracle label **y**
- Try current classifier **f(x)**
- If **f(x) = y**, celebrate
- If **f(x) ≠ y**, fix it

Online Training

- Get example **x** and oracle label **y**
- Try current classifier **f(x)** **f(x) =** sign(**$w_1x_1 + w_2x_2 + w_3x_4 +$** …)
- If **f(x) = y**, celebrate
- If **f(x) ≠ y**, fix it

Online Training

- Get example **x** and oracle label **y**
- Try current classifier **f(x)** **f(x) =** sign($\mathbf{w_1x_1 + w_2x_2 + w_3x_4 +}$ …)
- If **f(x) = y**, celebrate if **y = +1** and **f(x) = -1**, **w•x** is too small
- If **f(x) ≠ y**, fix it

Online Training

- Get example **x** and oracle label **y**
- Try current classifier **f(x)**
- If **f(x) = y**, celebrate
- If **f(x) ≠ y**, fix it

f(x) = sign(**$w_1x_1 + w_2x_2 + w_3x_4 +$** …)

if **y = +1** and **f(x) = -1**, **w•x** is too small

if **y = -1** and **f(x) = +1**, **w•x** is too big

How to Fix Your Broken Perceptron

How to Fix Your Broken Perceptron

w ← **w - x** if **f(x) = +1** and **y = -1**

How to Fix Your Broken Perceptron

w ← **w - x** if **f(x) = +1** and **y = -1**

w ← **w + x** if **f(x) = -1** and **y = +1**

How to Fix Your Broken Perceptron

w ← **w - x** if **f(x) = +1** and **y = -1**

w ← **w + x** if **f(x) = -1** and **y = +1**

w ← **w** if **f(x) = y**

How to Fix Your Broken Perceptron

w ← **w - x** if **f(x) = +1** and **y = -1**

w ← **w + x** if **f(x) = -1** and **y = +1**

w ← **w** if **f(x) = y**

w ← **w + yx** if **f(x) ≠ y**

How to Fix Your Broken Perceptron

w ← **w - x** if **f(x) = +1** and **y = -1**

w ← **w + x** if **f(x) = -1** and **y = +1**

w ← **w** if **f(x) = y**

w ← **w + yx** if **f(x) ≠ y**

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

f(x) = sign**(w$_1$ x$_1$ + w$_2$ x$_2$) = +1**

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

f(x) = sign**($w_1 x_1 + w_2 x_2$) = +1**

Example:

w = [2, 1], x = [3, 1], y = -1

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

f(x) = sign**($w_1 x_1 + w_2 x_2$) = +1**

Example:

w = [2, 1], x = [3, 1], y = -1

f(x) = sign(2(3) + 1(1))

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

f(x) = sign**($w_1 x_1 + w_2 x_2$) = +1**

Example:

w = [2, 1], x = [3, 1], y = -1

f(x) = sign(2(3) + 1(1))

Update:

W_1 = 2 - 3

W_2 = 1 - 1

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

f(x) = sign**($w_1 x_1 + w_2 x_2$) = +1**

Example:

w = [2, 1], x = [3, 1], y = -1

f(x) = sign(2(3) + 1(1))

Update:

W_1 = 2 - 3

W_2 = 1 - 1

w = [-1, 0]

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

f(x) = sign**($w_1 x_1 + w_2 x_2$) = +1**

Example:

w = [2, 1], x = [3, 1], y = -1

f(x) = sign(2(3) + 1(1))

Update:

W_1 = 2 - 3

W_2 = 1 - 1

w = [-1, 0]

New f(x) = -1(3) + 0(1)

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

y = +1, f(x) = -1

w • x

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

y = +1, f(x) = -1

w • x **(w + x) • x**

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

y = +1, f(x) = -1

$$\mathbf{w} \cdot \mathbf{x} \quad \leq \quad (\mathbf{w} + \mathbf{x}) \cdot \mathbf{x}$$

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

y = +1, f(x) = -1

$$w \cdot x \leq (w + x) \cdot x$$

$$w \cdot x + x \cdot x$$

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

y = +1, f(x) = -1

$$\mathbf{w} \cdot \mathbf{x} \leq (\mathbf{w} + \mathbf{x}) \cdot \mathbf{x}$$

$$\mathbf{w} \cdot \mathbf{x} + \mathbf{x} \cdot \mathbf{x}$$

y = -1, f(x) = +1

$$\mathbf{w} \cdot \mathbf{x} \geq (\mathbf{w} - \mathbf{x}) \cdot \mathbf{x}$$

How to Fix Your Broken Perceptron

~~**w ← w - x** if **f(x) = +1** and **y = -1**~~

~~**w ← w + x** if **f(x) = -1** and **y = +1**~~

w ← w if **f(x) = y**

w ← w + yx if **f(x) ≠ y**

y = +1, f(x) = -1

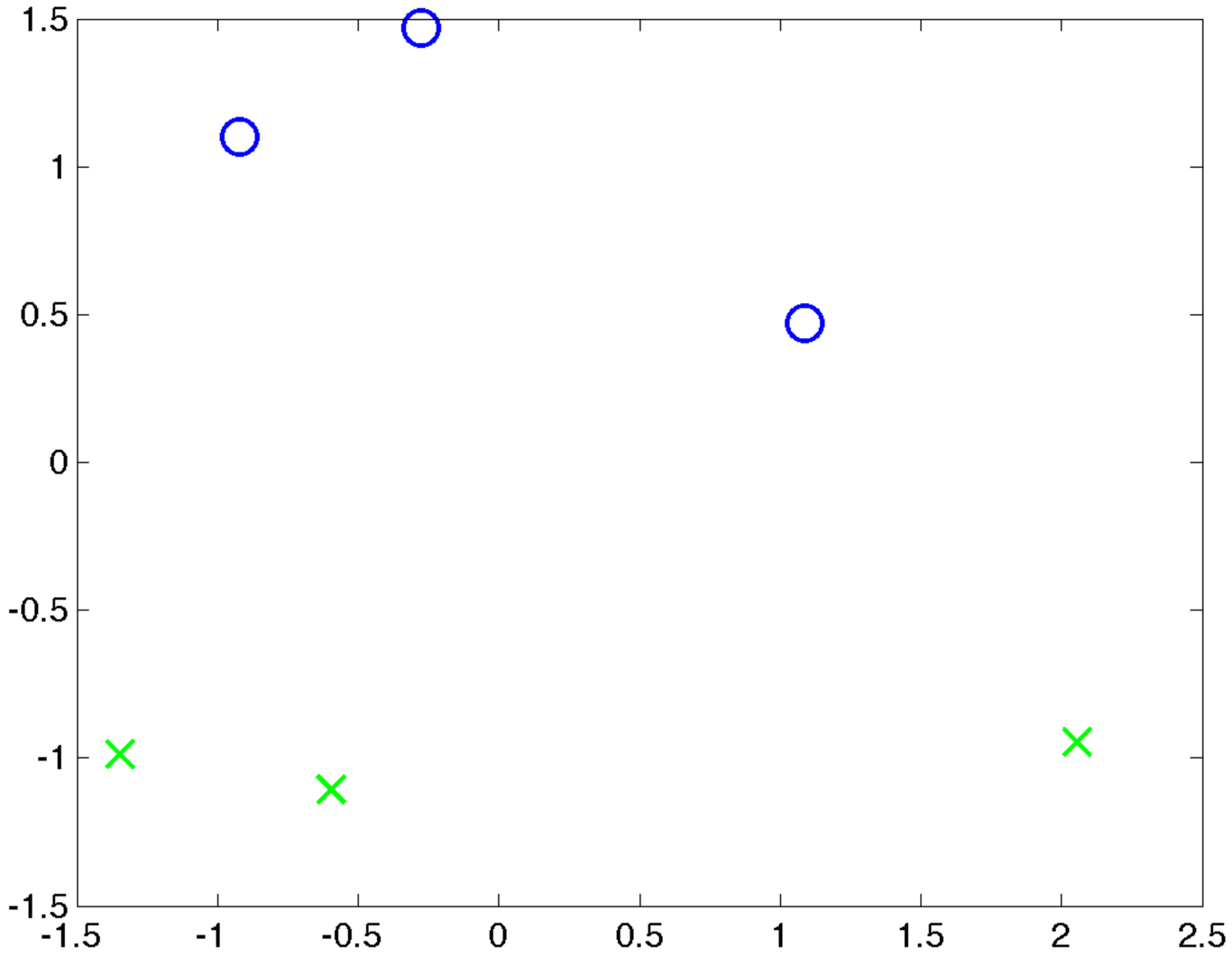
$$w \cdot x \leq (w + x) \cdot x$$

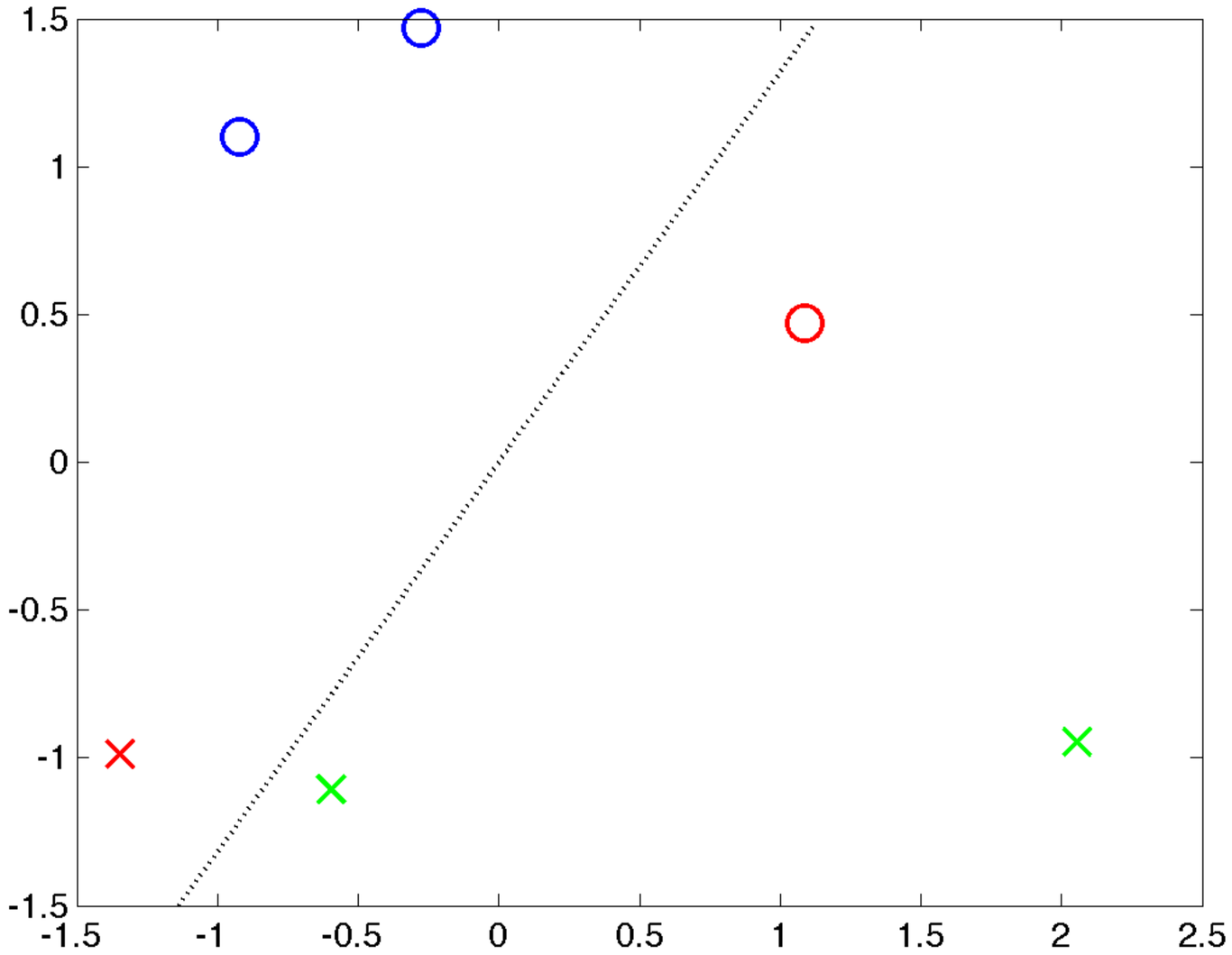
$$w \cdot x + x \cdot x$$

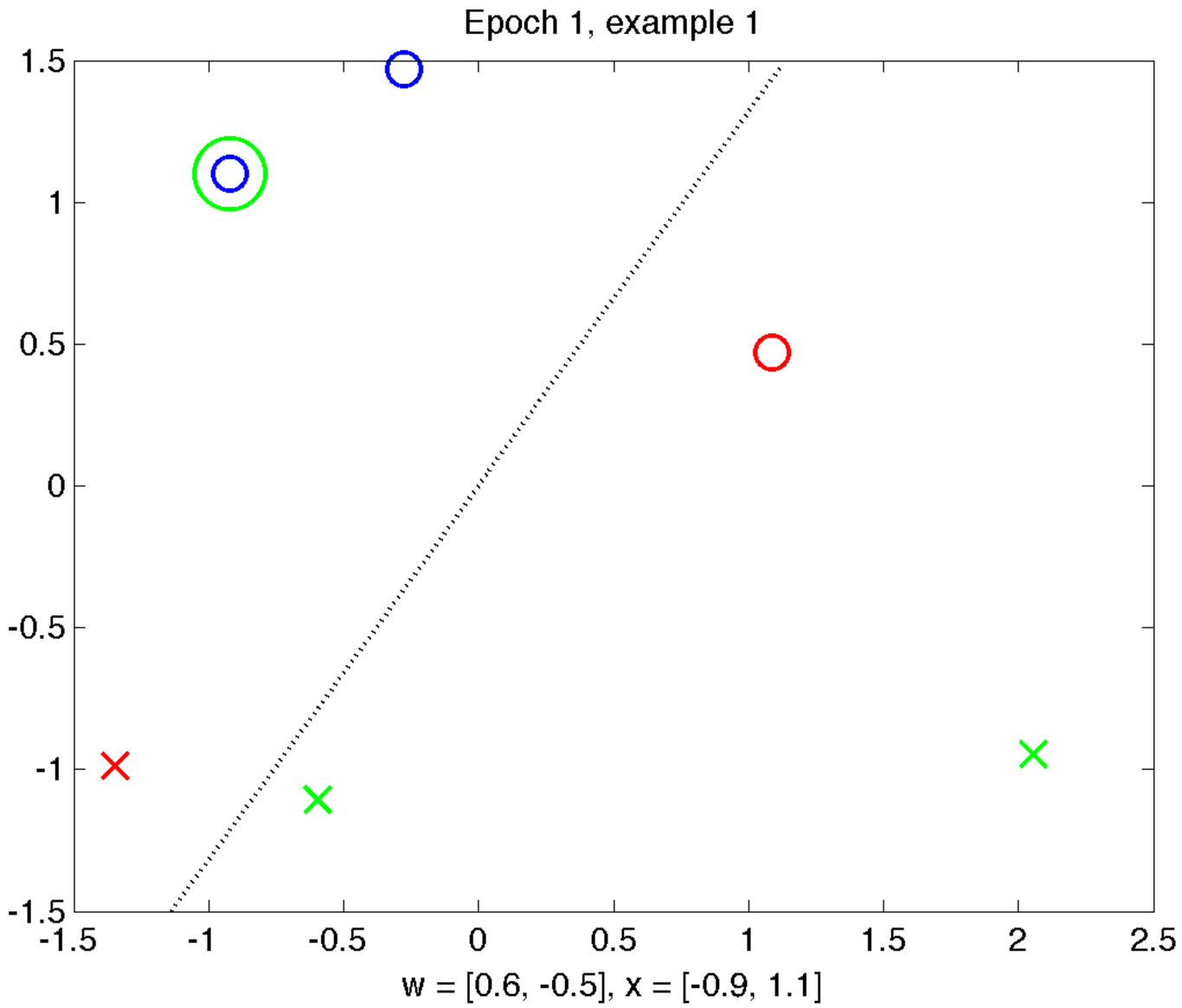
y = -1, f(x) = +1

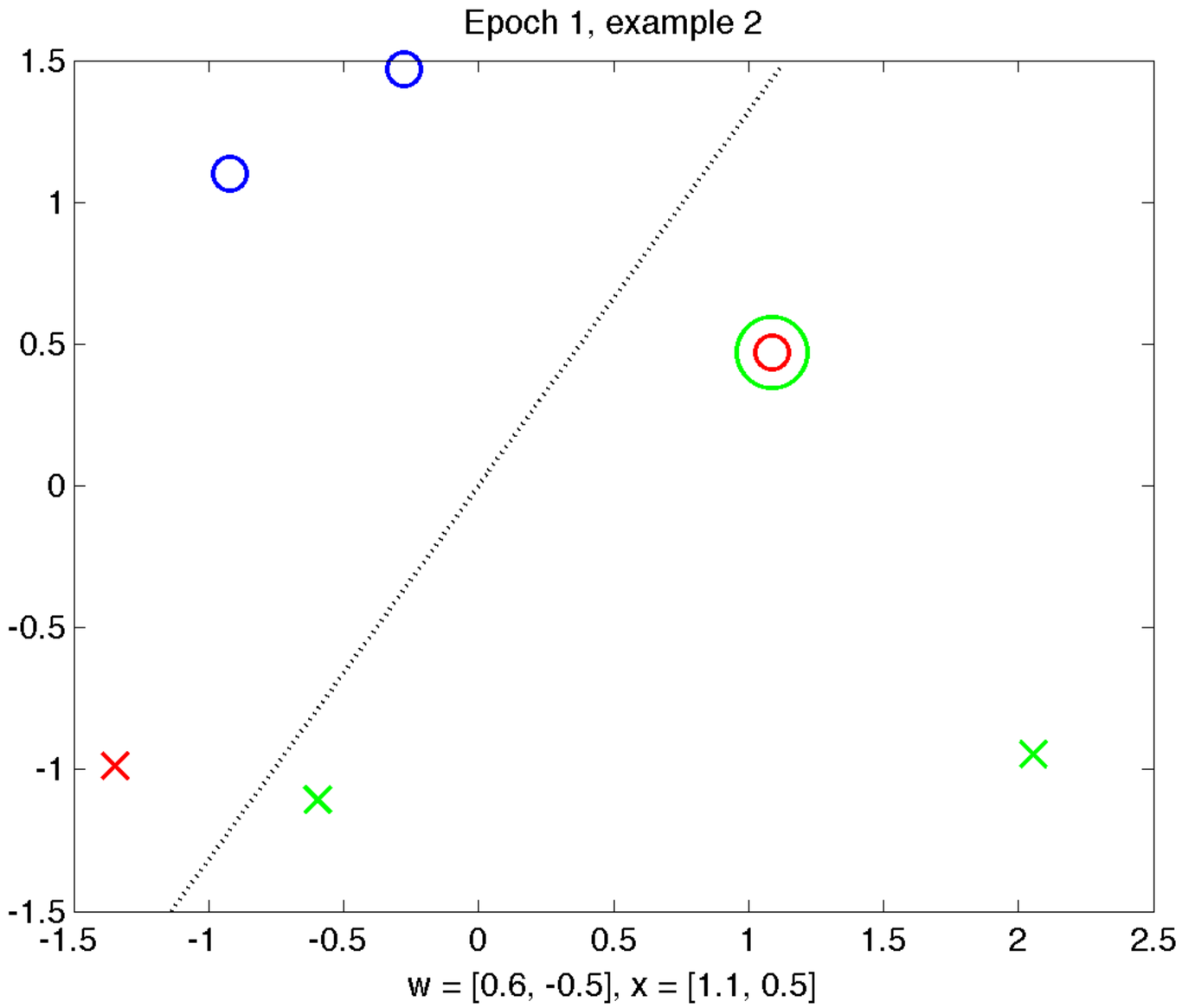
$$w \cdot x \geq (w - x) \cdot x$$

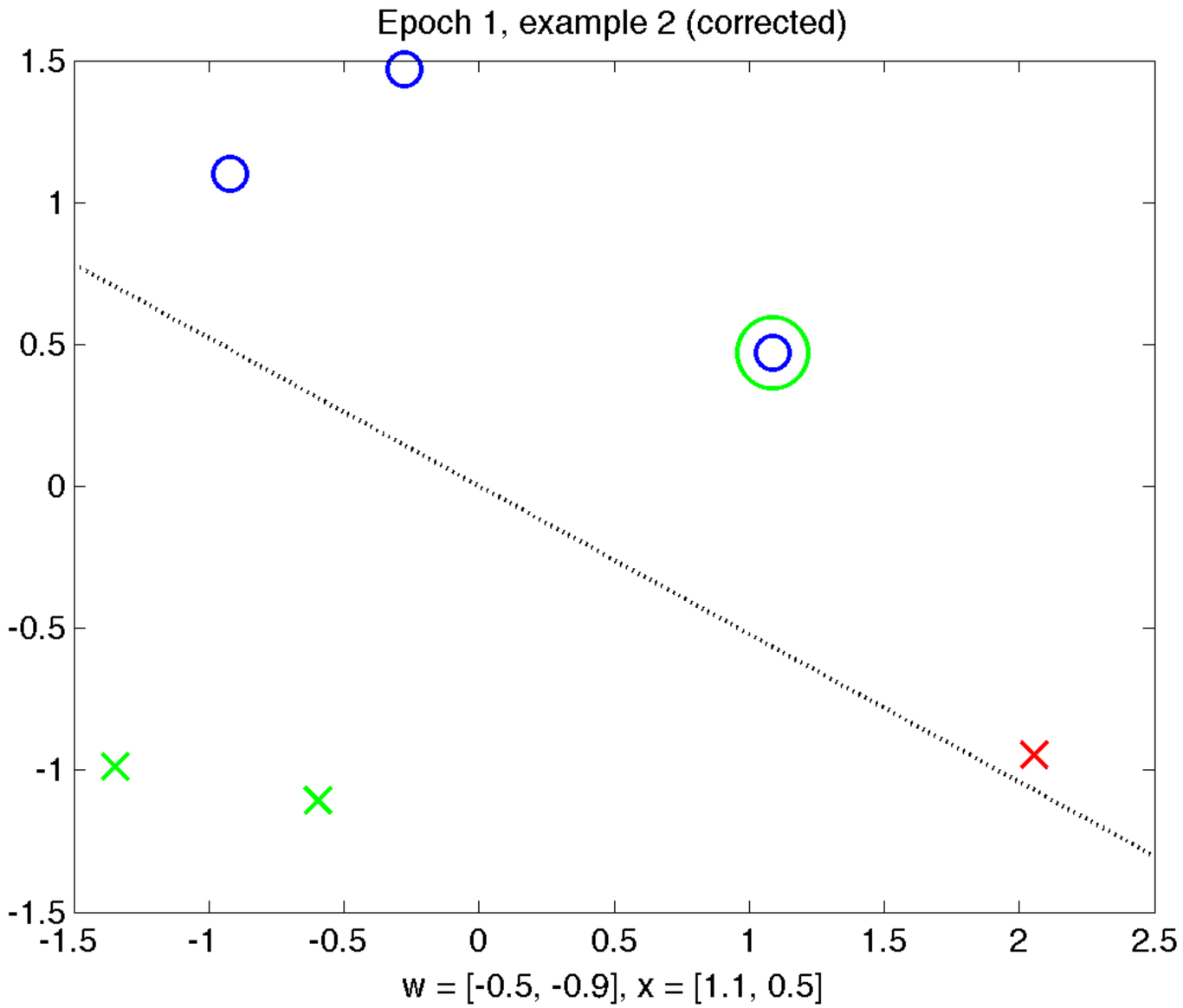
$$w \cdot x - x \cdot x$$

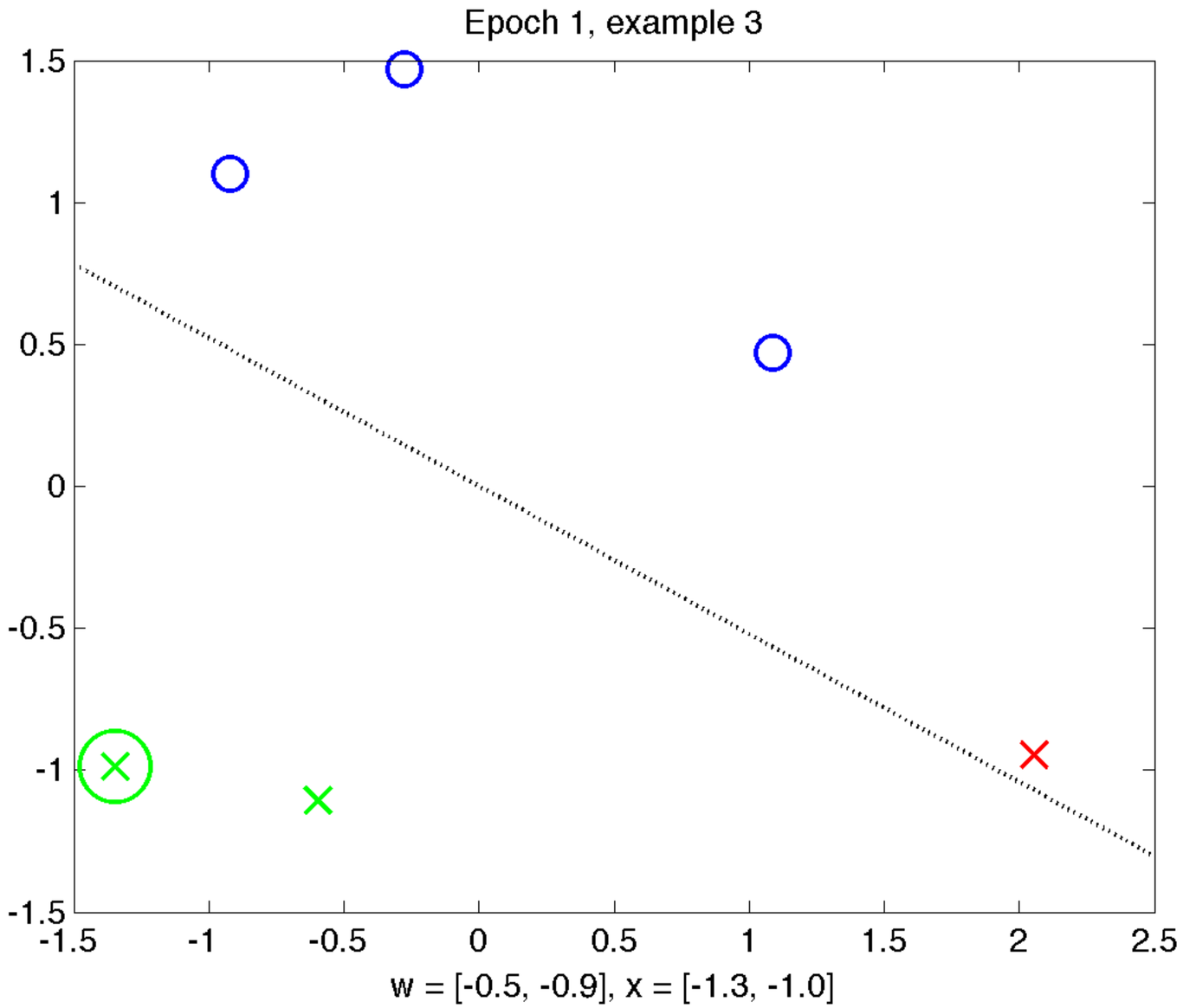

1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

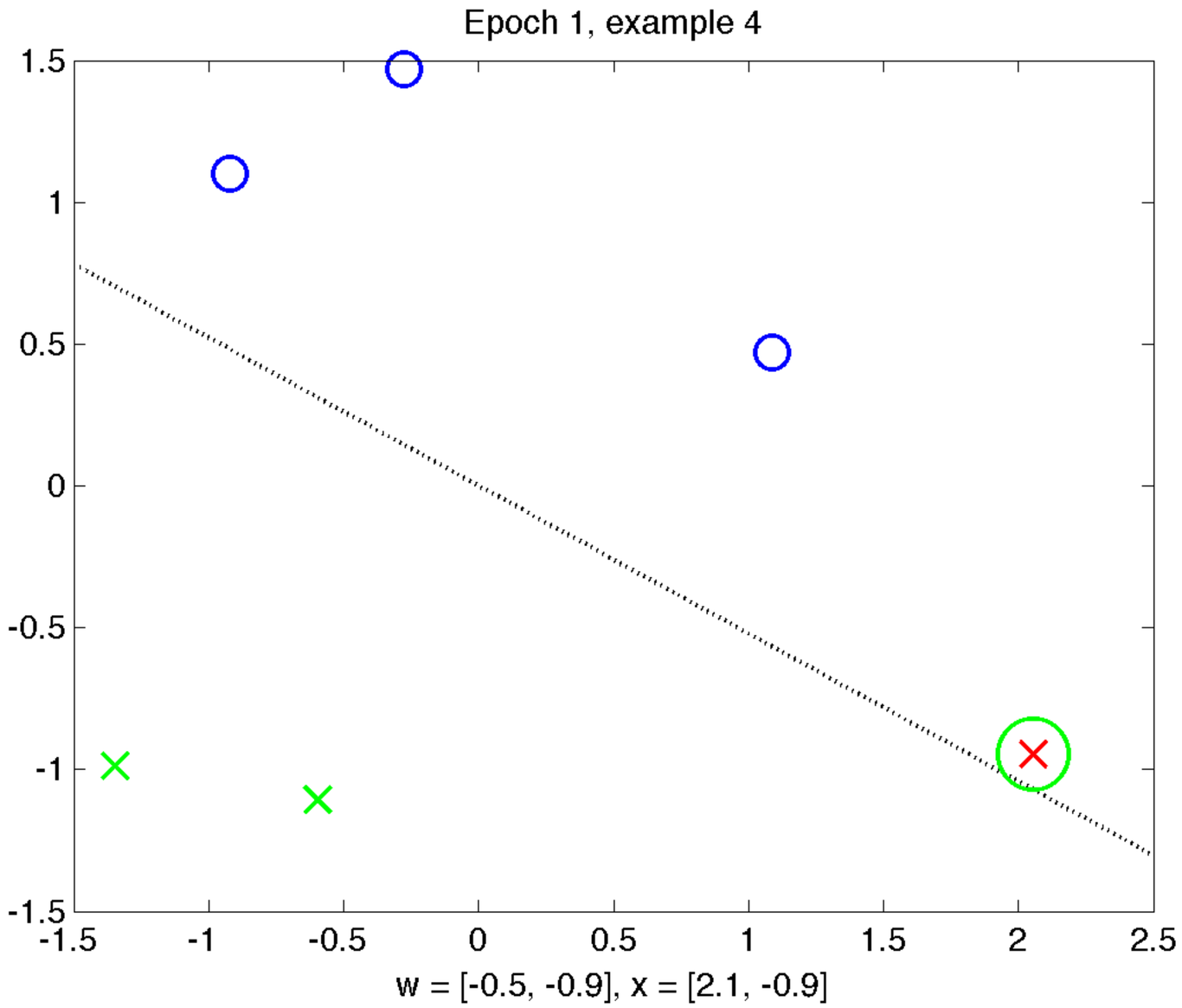

1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

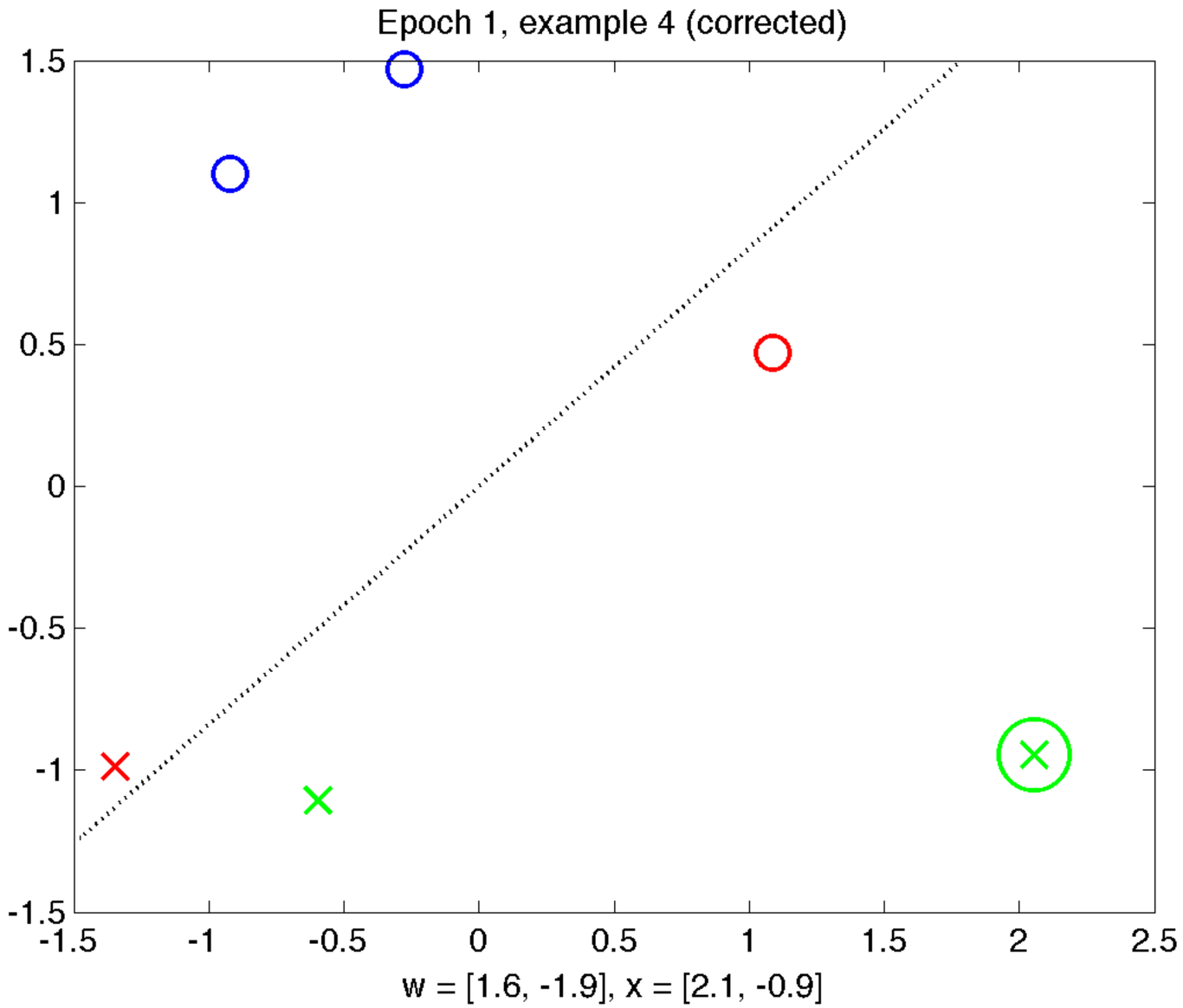

Epoch 1, example 1
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.6, -0.5], x = [-0.9, 1.1]

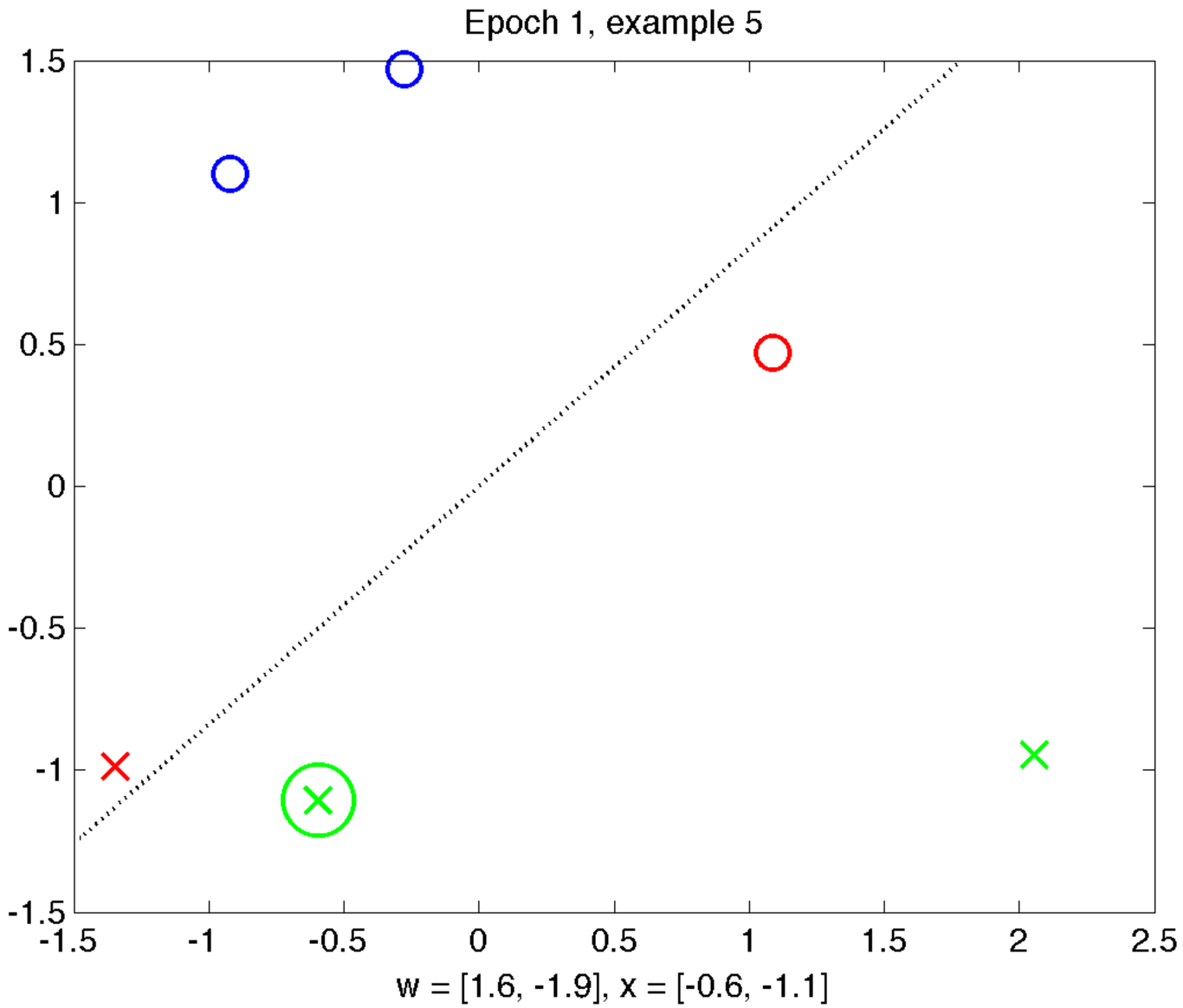

Epoch 1, example 2
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.6, -0.5], x = [1.1, 0.5]

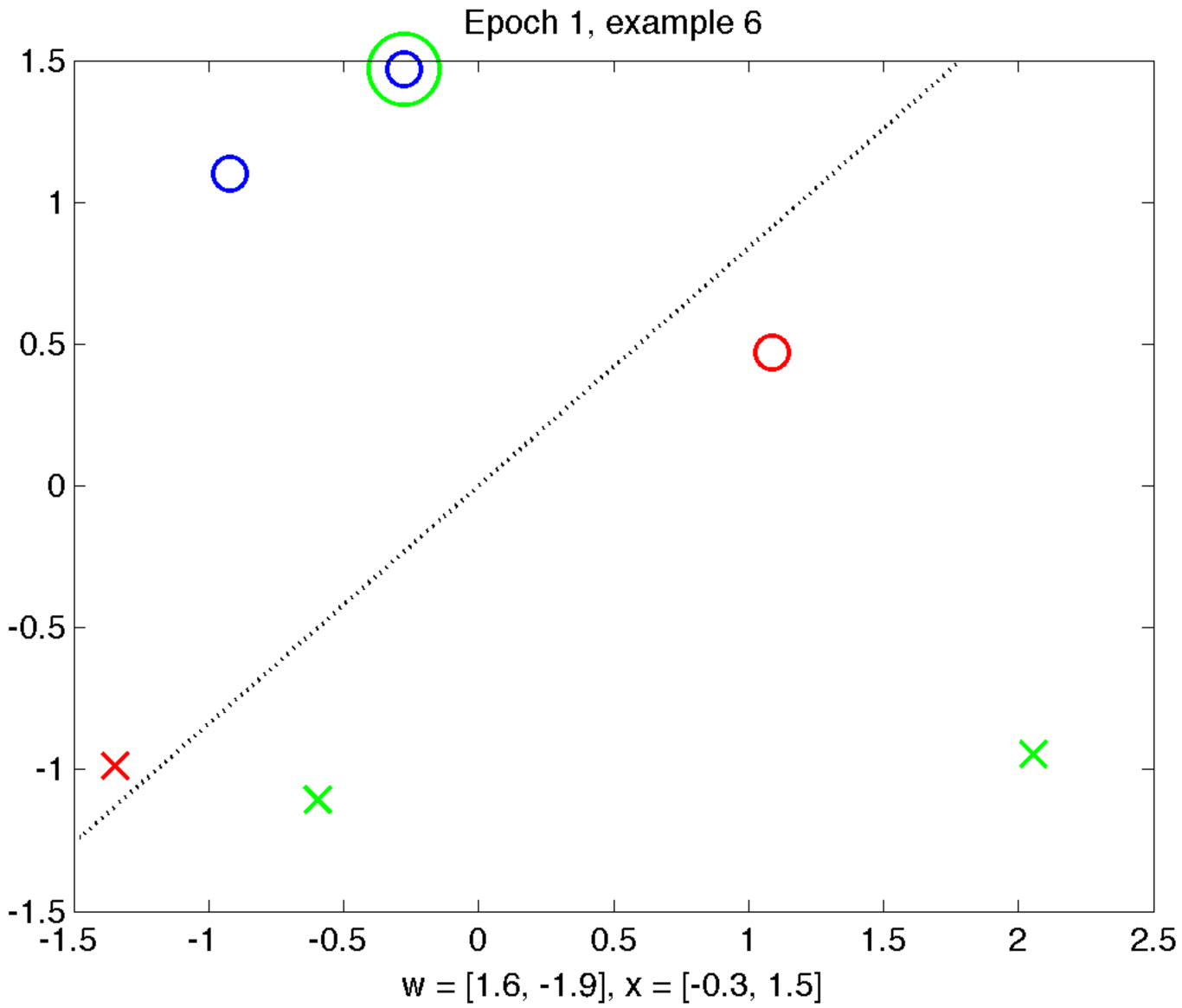

Epoch 1, example 2 (corrected)
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [-0.5, -0.9], x = [1.1, 0.5]

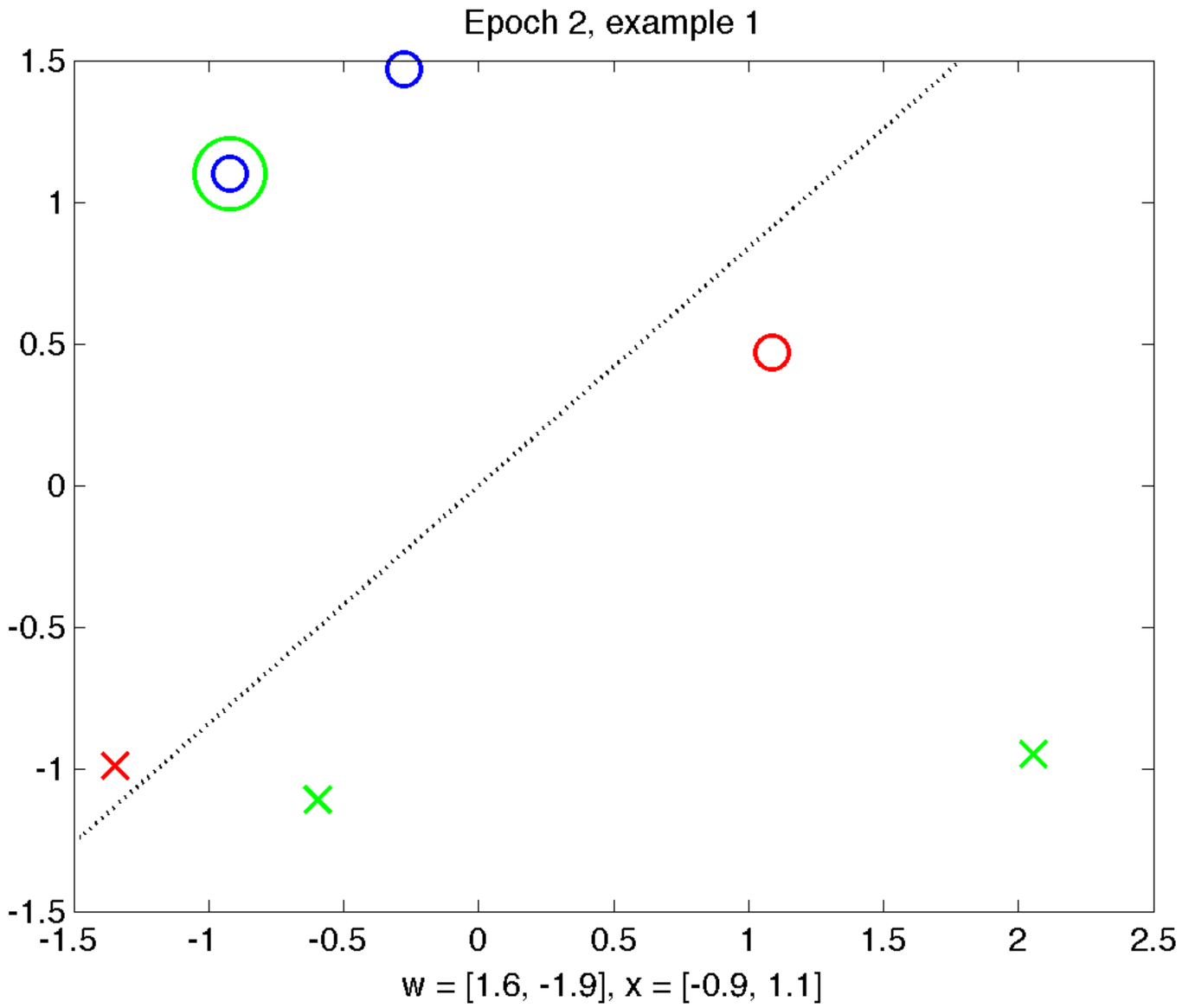

Epoch 1, example 3
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [-0.5, -0.9], x = [-1.3, -1.0]

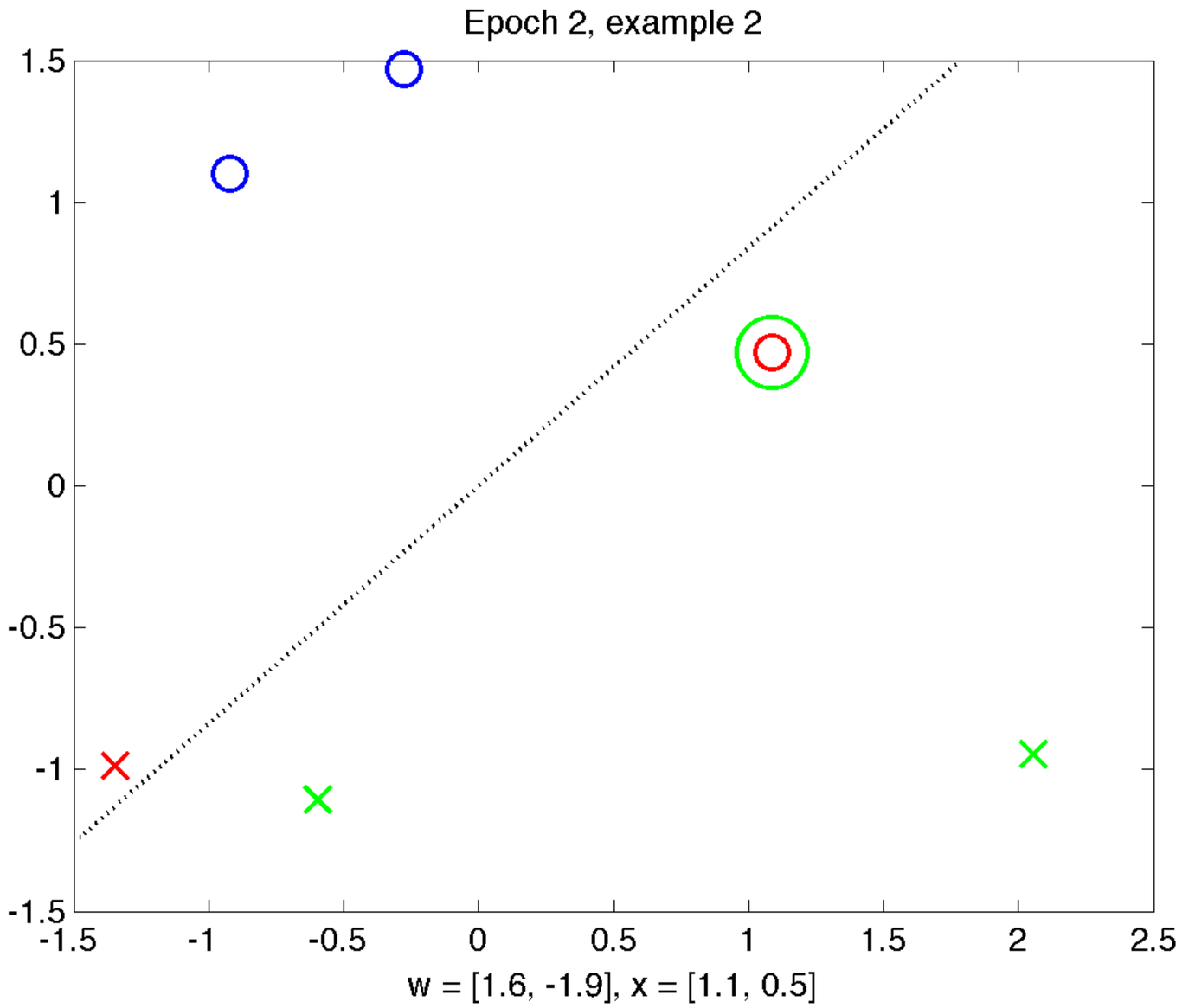

Epoch 1, example 4
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [-0.5, -0.9], x = [2.1, -0.9]

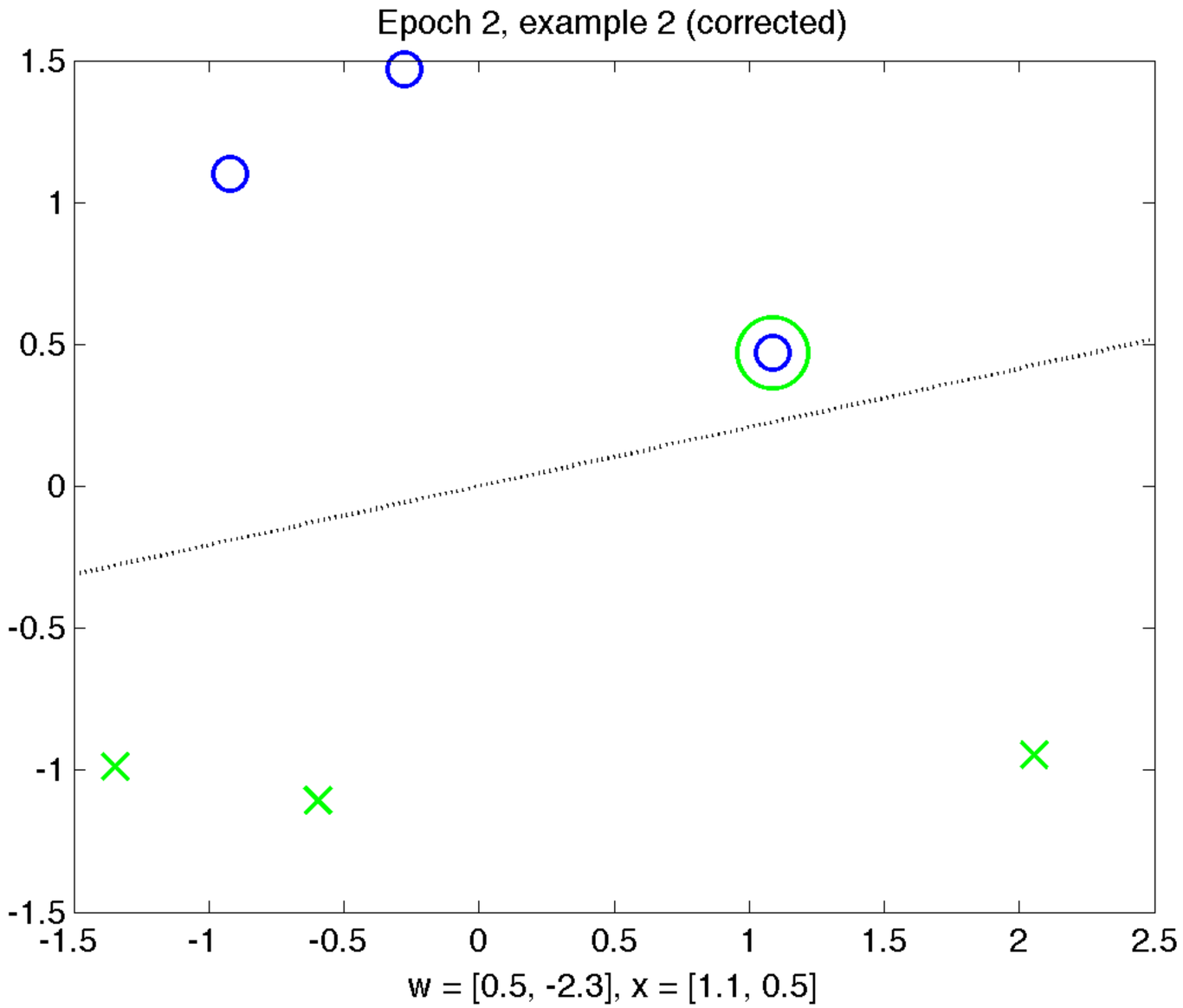

Epoch 1, example 4 (corrected)
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [1.6, -1.9], x = [2.1, -0.9]

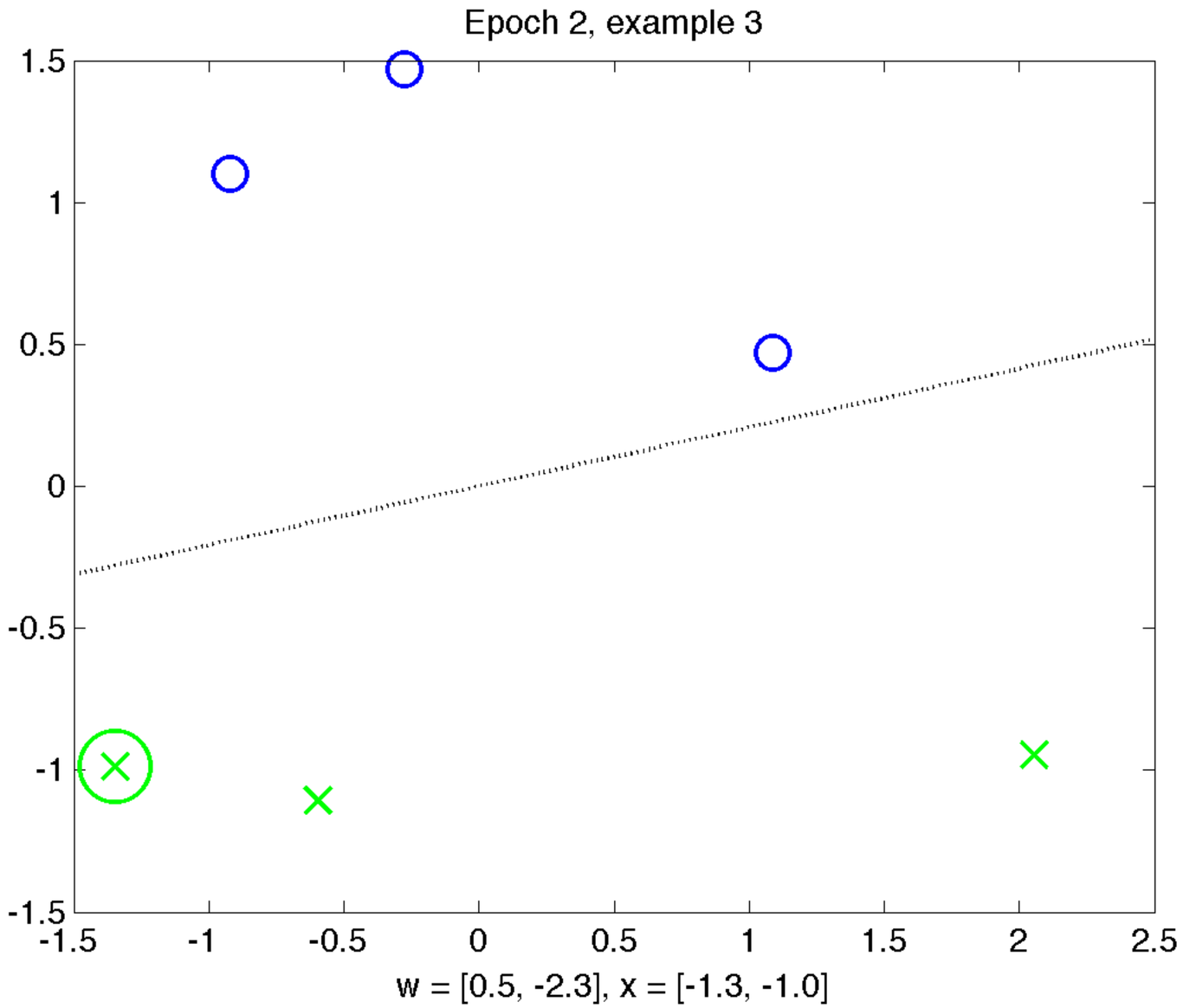

Epoch 1, example 5
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [1.6, -1.9], x = [-0.6, -1.1]

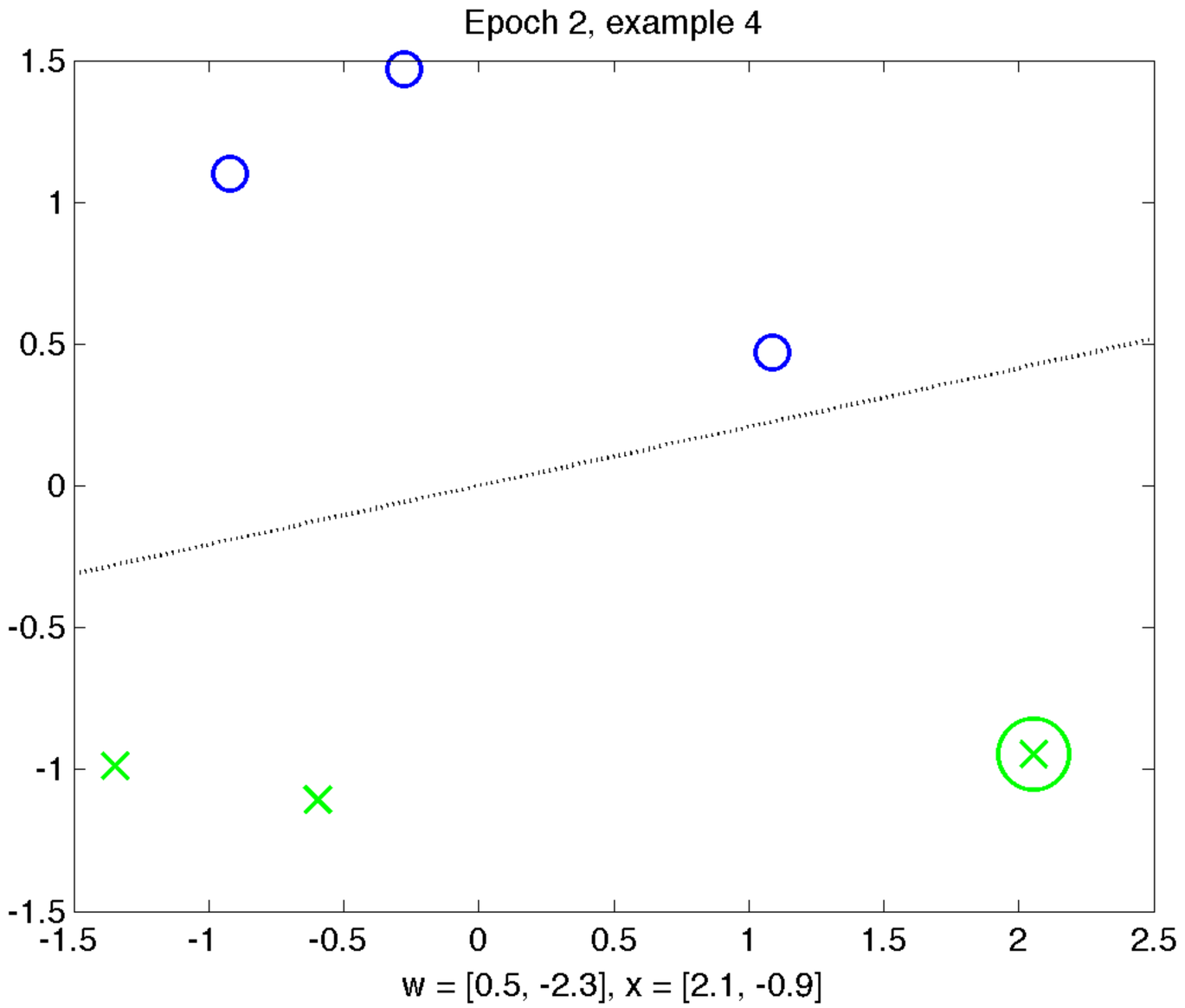

Epoch 1, example 6
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [1.6, -1.9], x = [-0.3, 1.5]

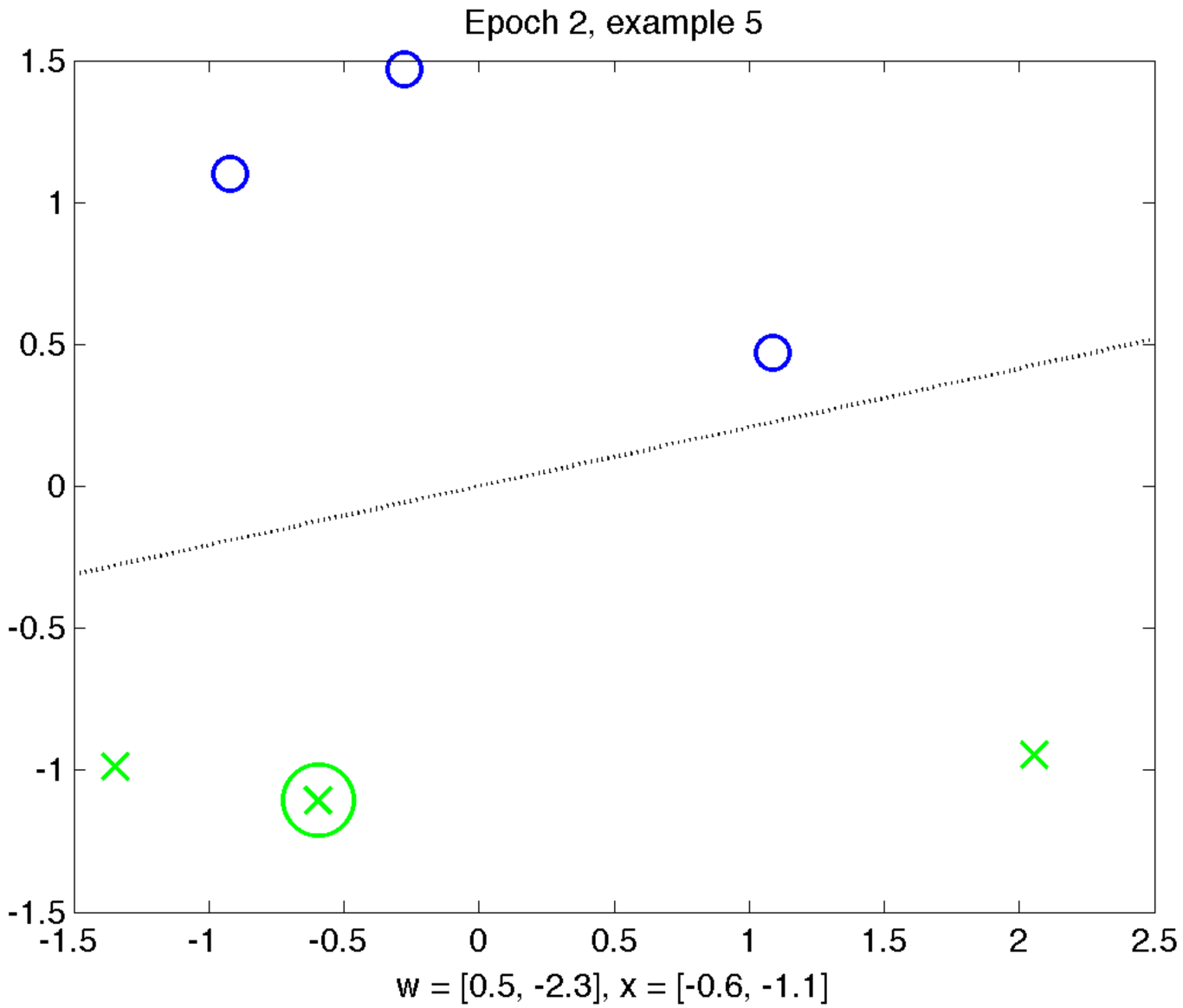

Epoch 2, example 1
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [1.6, -1.9], x = [-0.9, 1.1]

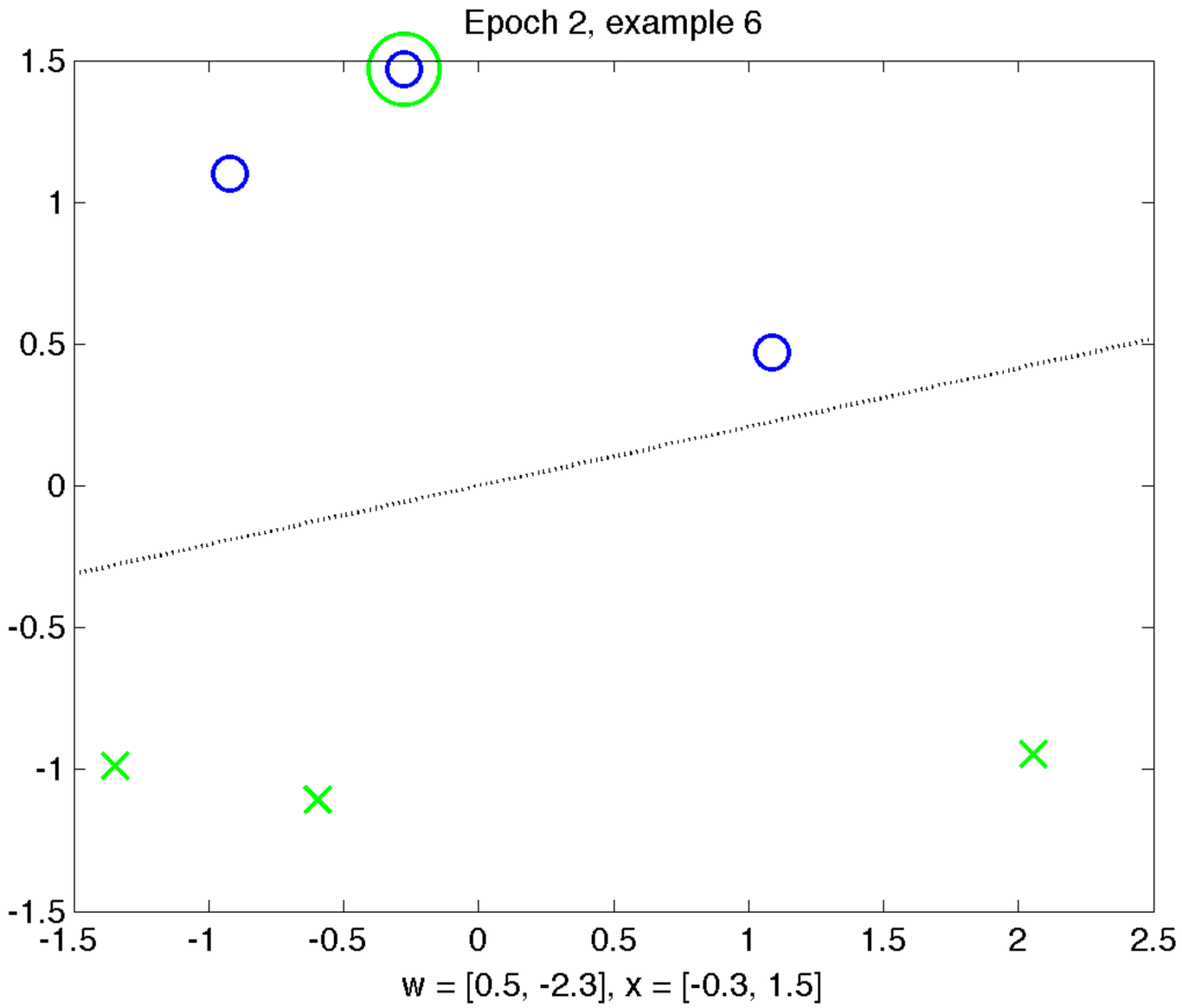

Epoch 2, example 2
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [1.6, -1.9], x = [1.1, 0.5]


Epoch 2, example 2 (corrected)
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.5, -2.3], x = [1.1, 0.5]


Epoch 2, example 3
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.5, -2.3], x = [-1.3, -1.0]


Epoch 2, example 4
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.5, -2.3], x = [2.1, -0.9]


Epoch 2, example 5
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.5, -2.3], x = [-0.6, -1.1]


Epoch 2, example 6
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
w = [0.5, -2.3], x = [-0.3, 1.5]

Binary Perceptron Learning

$$\mathbf{f(x)} = \text{sign}(\mathbf{w} \cdot \mathbf{x})$$

$$\mathbf{w} \leftarrow \mathbf{w} + \alpha \mathbf{y} \mathbf{x} \text{ if } \mathbf{f(x)} \neq \mathbf{y}$$

α = learning rate

Binary Perceptron Learning

$$\mathbf{f(x)} = \text{sign}(\mathbf{w} \cdot \mathbf{x})$$

$$\mathbf{w} \leftarrow \mathbf{w} + \alpha y \mathbf{x} \text{ if } \mathbf{f(x)} \neq y$$

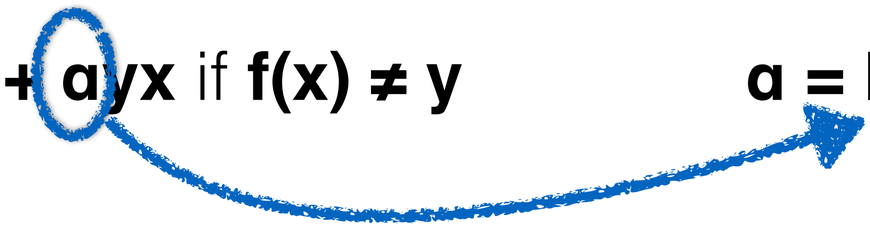
α = learning rate

Binary Perceptron Learning

sign**(w · x + b)** **bias**

f(x) = ~~sign**(w · x)**~~

w ← w + **αyx** if **f(x) ≠ y** **α =** learning rate



Binary Perceptron Learning

sign**(w · x + b)** **bias**

f(x) = ~~sign**(w · x)**~~

w ← w + αyx if **f(x) ≠ y** **α =** learning rate

b ← b + αy if **f(x) ≠ y**

Multiclass Output

-1

+1

Multiclass Output

Multiclass Perceptron

Multiclass Perceptron

- Separate weights for each class: $\mathbf{w^0}$, $\mathbf{w^1}$, $\mathbf{w^2}$, $\mathbf{w^3}$, $\mathbf{w^4}$, $\mathbf{w^5}$, $\mathbf{w^6}$, ...

Multiclass Perceptron

- Separate weights for each class: $\mathbf{w^0, w^1, w^2, w^3, w^4, w^5, w^6, \ldots}$
- Predict according to maximum scoring class:

$$\mathbf{f(x) = \underset{y'}{\text{argmax}}\, w^{y'} \cdot x}$$

Multiclass Perceptron

- Separate weights for each class: $\mathbf{w^0, w^1, w^2, w^3, w^4, w^5, w^6}$, ...
- Predict according to maximum scoring class:

$$\mathbf{f(x) = \underset{y'}{\text{argmax}}\, w^{y'} \cdot x}$$

- Learning update:

$$\mathbf{w^y \leftarrow w^y + x}$$

$$\mathbf{w^{f(x)} \leftarrow w^{f(x)} - x}$$

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

x

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

x

y = 2

Multiclass Perceptron

- Learning update:

$$\mathbf{w^y \leftarrow w^y + x}$$

$$\mathbf{w^{f(x)} \leftarrow w^{f(x)} - x}$$

x

y = 2

f(x) = 6

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

x

y = 2

f(x) = 6

$\mathbf{w}^{2} \leftarrow \mathbf{w}^{2} + \mathbf{x}$ (make $\mathbf{w}^{2} \cdot \mathbf{x}$ greater)

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

x

y = 2

f(x) = 6

$\mathbf{w}^{2} \leftarrow \mathbf{w}^{2} + \mathbf{x}$ (make $\mathbf{w}^{2} \cdot \mathbf{x}$ greater)

$\mathbf{w}^{6} \leftarrow \mathbf{w}^{6} - \mathbf{x}$ (make $\mathbf{w}^{6} \cdot \mathbf{x}$ lesser)

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

x

y = 2

Multiclass Perceptron

- Learning update:

$$\mathbf{w}^{y} \leftarrow \mathbf{w}^{y} + \mathbf{x}$$

$$\mathbf{w}^{f(x)} \leftarrow \mathbf{w}^{f(x)} - \mathbf{x}$$

x

y = 2

f(x) = 2

Multiclass Perceptron

- Learning update:

$$\mathbf{w^y \leftarrow w^y + x}$$

$$\mathbf{w^{f(x)} \leftarrow w^{f(x)} - x}$$

x

y = 2

f(x) = 2

$$\mathbf{w^2 \leftarrow w^2 + x}$$

Multiclass Perceptron

- Learning update:

$$\mathbf{w^y \leftarrow w^y + x}$$

$$\mathbf{w^{f(x)} \leftarrow w^{f(x)} - x}$$

x

y = 2

f(x) = 2

$$\mathbf{w^2 \leftarrow w^2 + x}$$

$$\mathbf{w^2 \leftarrow w^2 - x}$$

Multiclass Perceptron

- Learning update:

$$\mathbf{w^y \leftarrow w^y + x}$$

$$\mathbf{w^{f(x)} \leftarrow w^{f(x)} - x}$$

x

y = 2

f(x) = 2

$$\mathbf{w^2 \leftarrow w^2 + x - x}$$

Multiclass Perceptron

- Learning update:

$$\mathbf{w^y \leftarrow w^y + x}$$

$$\mathbf{w^{f(x)} \leftarrow w^{f(x)} - x}$$

x

y = 2

f(x) = 2

$$\mathbf{w^2 \leftarrow w^2 + x - x = w^2}$$

Perceptron Summary

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- Linear classifier multiplies **weights** by input **features**

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- Multiclass: use **multiple weight vectors**, choose max scoring

Perceptron Summary

- Linear classifier multiplies **weights** by input **features**
- Learn by updating when wrong
  - If score too low, make score higher
  - If score too high, make score lower
- Multiclass: use **multiple weight vectors**, choose max scoring
  - Learn by adjusting score of predicted class and true class