Bayes Net Inference

Machine Learning
Bert Huang

Inference

- Given a Bayesian Network describing P(X, Y, Z), what is P(Y)
  - First approach: **enumeration**

$$P(R, W, S, C) = P(R)\, P(C)\, P(W \mid C, R)\, P(S \mid W)$$

$$P(r|s) = \sum_w \sum_c P(r, w, s, c)/P(s)$$

$$P(r|s) \propto \sum_w \sum_c P(r)P(c)P(w|c, r)P(s|w)$$

$$P(r|s) \propto P(r) \sum_w P(s|w) \sum_c P(c)P(w|c, r)$$

Second Approach: Variable Elimination

$$P(r|s) \propto \sum_w \sum_c P(r)P(c)P(w|c,r)P(s|w)$$

$$f_C(w) = \sum_c P(c)P(w|c,r)$$

$$P(r|s) \propto \sum_w P(r)P(s|w)f_c(w)$$

$$P(W, X, Y, Z) = P(W)P(X|W)P(Y|X)P(Z|Y)$$

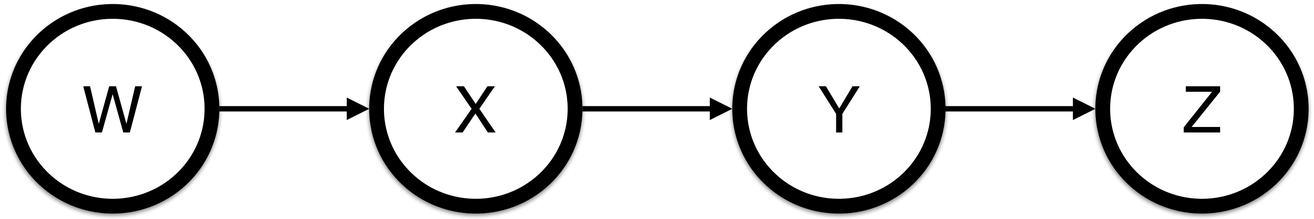
$P(Y)$?

$$P(Y) = \sum_w \sum_x \sum_z P(w)P(x|w)P(Y|x)P(z|Y)$$

$$f_w(x) = \sum_w P(w)P(x|w)$$

$$P(Y) = \sum_x \sum_z f_w(x)P(Y|x)P(z|Y)$$

$$f_x(Y) = \sum_x f_w(x)P(Y|x) \qquad P(Y) = \sum_z f_x(Y)P(z|Y)$$



$$P(Y) = \sum_w \sum_x \sum_z P(w)P(x|w)P(Y|x)P(z|Y)$$

$$f_w(x) = \sum_w P(w)P(x|w)$$

$$P(Y) = \sum_x \sum_z f_w(x)P(Y|x)P(z|Y)$$

$$f_x(Y) = \sum_x f_w(x)P(Y|x)$$

$$P(Y) = \sum_z f_x(Y)P(z|Y)$$

General Inference: Variable Elimination

- Every variable that is not an ancestor of a query variable or evidence variable is irrelevant to the query. Sum out irrelevant variables.
- Iterate:
  - choose variable to eliminate
  - sum terms relevant to variable, generate new factor
  - until no more variables to eliminate
- Exact inference is #P-Hard
  - in tree-structured BNs, linear time (in number of table entries)