CS5804 Homework 4 Written Problem

Homework must be submitted electronically following the instructions on the course homepage. Make sure to explain you reasoning or show your derivations. Except for answers that are especially straightforward, you will lose points for unjustified answers, even if they are correct.

1. (Problem 14.6 in R+N) Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism: that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

   (a) (1 point) Which of the three networks in Figure 1 claim that

   $$P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})?$$

   (b) (2 points) Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

   (c) (1 point) Which of the three networks is the best description of the hypothesis?

   (d) (3 points) Write down the conditional probability table for the G_{child} node in network (a), in terms of s and m.

   (e) (4 points) Suppose that $P(G_{father} = l) = P(G_{mother} = l) = q$. In network (a), derive an expression for $P(G_{child} = l)$ in terms of m and q only, by conditioning on its parent nodes.

   (f) (4 points) Under conditions of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this fact to calculate the value of q, and, given what you know about handedness in humans (e.g., that left-handedness is much less common than right-handedness), explain why the hypothesis described at the beginning of this question must be wrong.
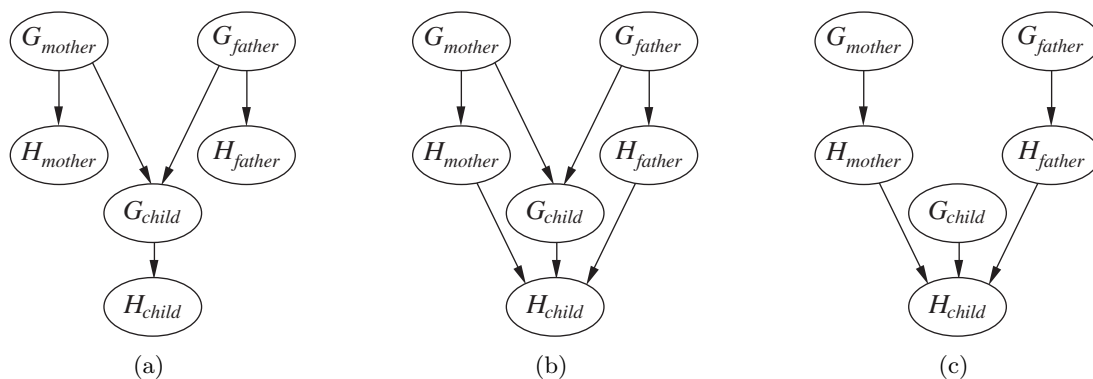

Figure 1: Possible Bayesian network representations of the genetics of handedness.