Program Static Analysis

Overview

- Program static analysis
- Abstract interpretation
- Static analysis techniques

What is static analysis?

- The analysis to understand computer software without executing programs
  - Simple coding style
    - Empty statement, EqualsHashcode
  - Complex property of the program
    - the program's implementation matches its specification
  - "Given program P and specification S, does P satisfy S ?"
- Can be conducted on source code or object code

Has anyone done static analysis?

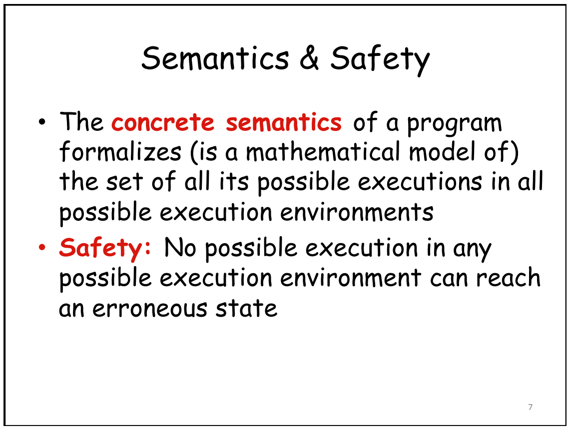
- Code review
- …

Why static analysis?

- Program comprehension
  - Is this value a constant?
- Bug finding
  - Is a file closed on every path after all its access?
- Program optimization
  - Constant propagation

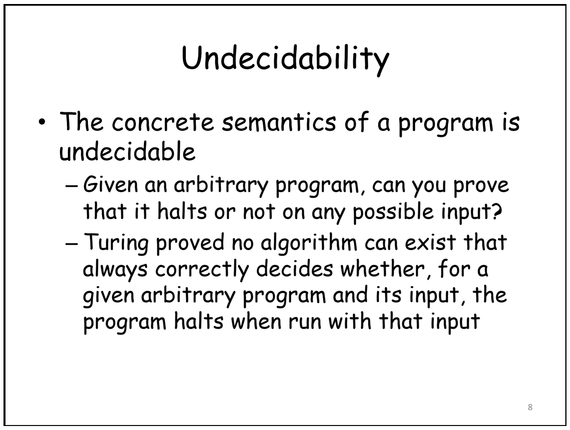
An Informal Introduction to Abstract Interpretation

Patrick Cousot[2]
Modified by Na Meng



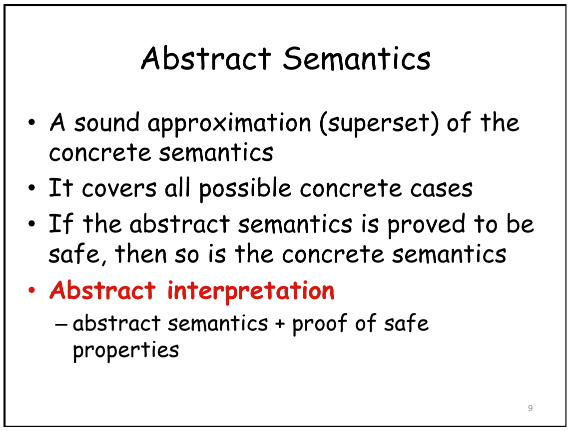
Semantics & Safety

- The **concrete semantics** of a program formalizes (is a mathematical model of) the set of all its possible executions in all possible execution environments
- **Safety:** No possible execution in any possible execution environment can reach an erroneous state



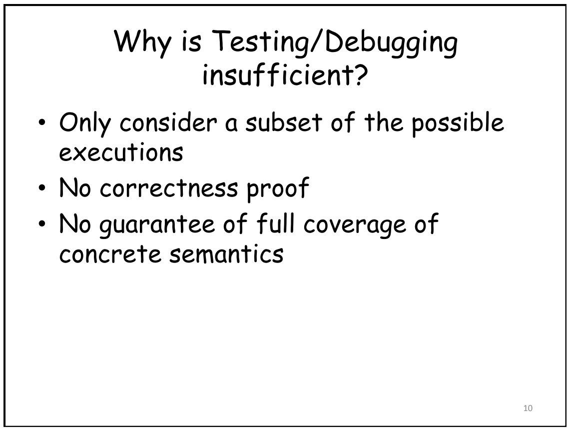
Undecidability

- The concrete semantics of a program is undecidable
  - Given an arbitrary program, can you prove that it halts or not on any possible input?
  - Turing proved no algorithm can exist that always correctly decides whether, for a given arbitrary program and its input, the program halts when run with that input



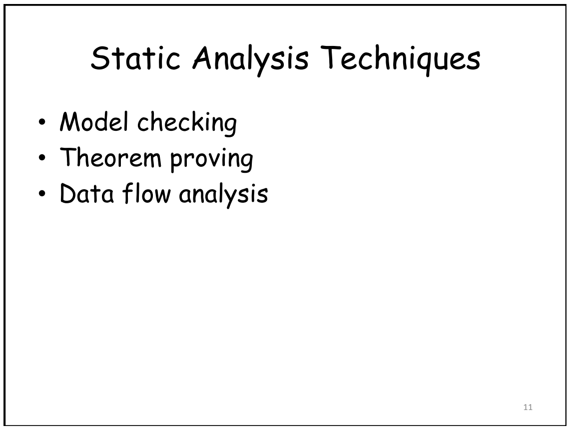
Abstract Semantics

- A sound approximation (superset) of the concrete semantics
- It covers all possible concrete cases
- If the abstract semantics is proved to be safe, then so is the concrete semantics
- **Abstract interpretation**
  - abstract semantics + proof of safe properties



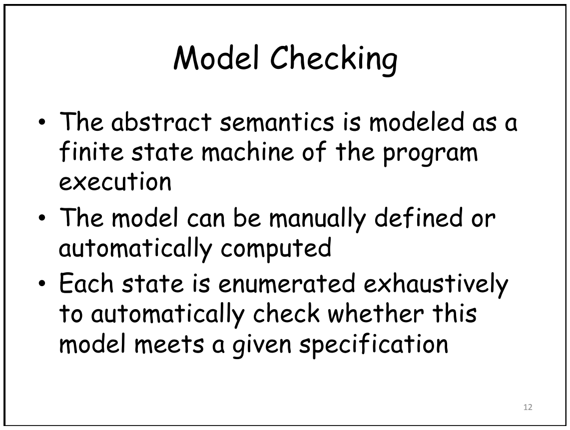
Why is Testing/Debugging insufficient?

- Only consider a subset of the possible executions
- No correctness proof
- No guarantee of full coverage of concrete semantics



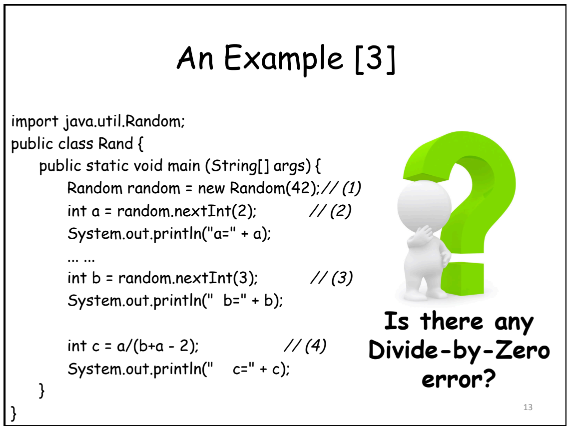
Static Analysis Techniques

- Model checking
- Theorem proving
- Data flow analysis



Model Checking

- The abstract semantics is modeled as a finite state machine of the program execution
- The model can be manually defined or automatically computed
- Each state is enumerated exhaustively to automatically check whether this model meets a given specification

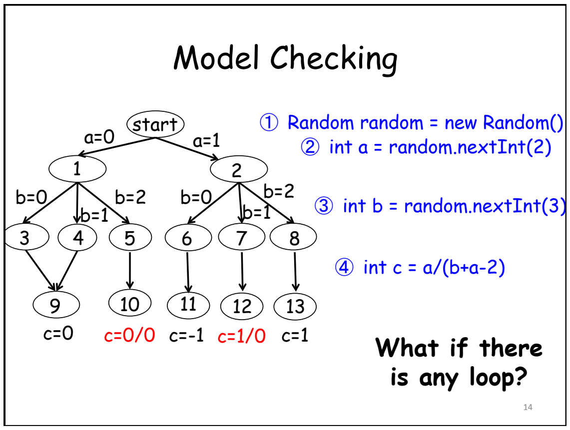


An Example [3]

```
import java.util.Random;
public class Rand {
      public static void main (String[] args) {
            Random random = new Random(42);// (1)
            int a = random.nextInt(2);          // (2)
            System.out.println("a=" + a);
            ... ...
            int b = random.nextInt(3);          // (3)
            System.out.println("  b=" + b);

            int c = a/(b+a - 2);                // (4)
            System.out.println("    c=" + c);
      }
}
```

Is there any Divide-by-Zero error?



Model Checking

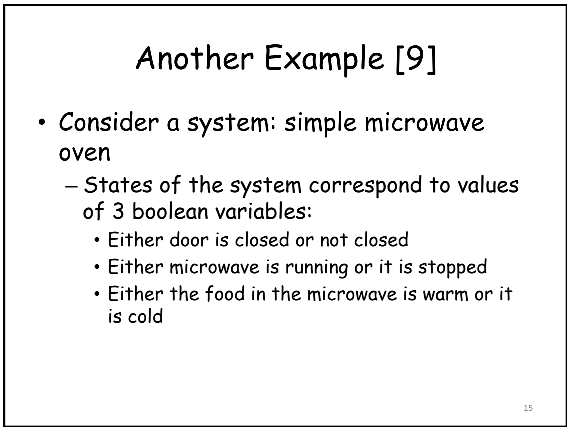
① Random random = new Random()

② int a = random.nextInt(2)

③ int b = random.nextInt(3)

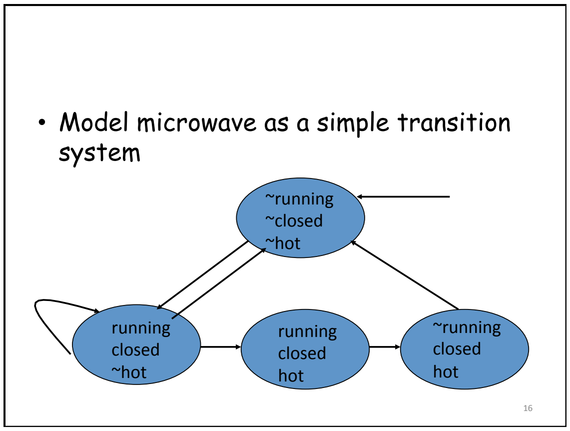
④ int c = a/(b+a-2)

What if there is any loop?

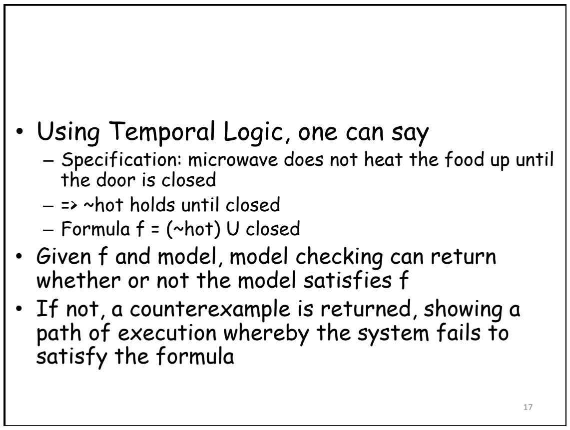


Another Example [9]

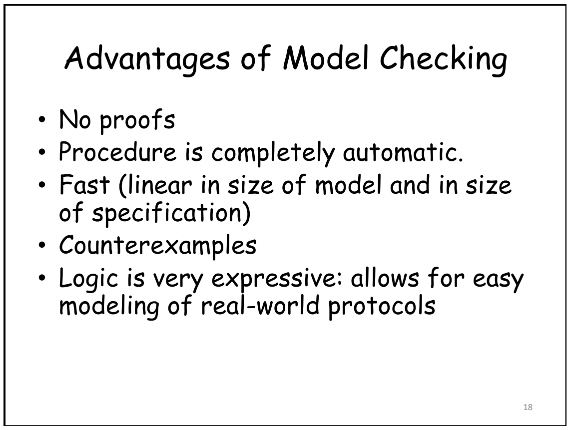
- Consider a system: simple microwave oven
  - States of the system correspond to values of 3 boolean variables:
    - Either door is closed or not closed
    - Either microwave is running or it is stopped
    - Either the food in the microwave is warm or it is cold



- Model microwave as a simple transition system



- Using Temporal Logic, one can say
  - Specification: microwave does not heat the food up until the door is closed
  - => ~hot holds until closed
  - Formula f = (~hot) U closed
- Given f and model, model checking can return whether or not the model satisfies f
- If not, a counterexample is returned, showing a path of execution whereby the system fails to satisfy the formula



Advantages of Model Checking

- No proofs
- Procedure is completely automatic.
- Fast (linear in size of model and in size of specification)
- Counterexamples
- Logic is very expressive: allows for easy modeling of real-world protocols

Disadvantages of Model Checking

- There can be too many states to enumerate (state explosion)
- Abstract model creation puts burden on programmers
- The model may be wrong
  - If verification fails, is the problem in the model or the program?

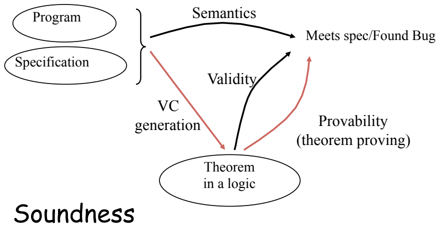
Solutions to Space Explosion

- 1987: Ken McMillan developed a symbolic model checking approach where the system was represented using Binary Decision Diagrams
  - Data structure for representing boolean functions
  - Concise representations for transition systems, fast manipulation
  - Good for synchronous systems
- Partial Order Reduction: reduce number of states that must be explicitly enumerated
  - Good for asynchronous systems

Today's Model Checkers

- Can handle systems with between 100 and 300 state variables
- Systems with 10^{120} reachable states have been checked!
- Using appropriate abstraction techniques, systems with an essentially unbounded number of states can be checked

Theorem Proving [4]



- Soundness
  - If the theorem is valid then the program meets specification
  - If the theorem is provable then it is valid

- From programs and specs to theorems
  - Verification condition generation
- From theorems to proofs
  - Theorem provers

Verification Condition Generation

- State predicates/assertions: Boolean functions on program states
  - E.g., x = 8, x < y, true, false
- You can deduce verification condition predicates from known predicates at a given program location

Hoare Triples [6]

- For any predicates P and Q and program S,

postcondition

{P} S {Q}

precondition

says that if S is started in a state satisfying P, then it terminates in Q

  - E.g., {true} x := 12 {x = 12}, {x < 40} x := x+1 {x ≤ 40}

Precise Triples

- If {P} S {Q ∧ R} holds, then do {P} S {Q} and {P} S {R} hold?
- Strongest postcondition
  - The most precise postcondition (Q ∧ R), which implies any postcondition satisfied by the final state of any execution x of S

$$\forall x, sp(S, P) \Rightarrow Q$$

  - E.g., {true} x := 12 {x = 12} vs. {true} x:=12 {x > 0}, which postcondition is stronger?

Precise Triples

- If {P} S {R} or {Q} S {R} hold, then does {P ∨ Q} S {R} hold?
- Weakest preconditions
  - The most general precondition {P ∨ Q}, is the "weakest" precondition on the initial state ensuring that execution of S terminates in a final state satisfying R.

$$\forall x, P \Rightarrow wp(S, R)$$

  - E.g., {x=13} x = x+3 {x >13} vs. {x>10} x = x+3 {x >13}, which precondition is weaker?

Example: Does the program satisfy the specification?

- Specification

requires true (precondition)

ensures c = a ∨ b (postcondition)

- Program

```
bool or(bool a, bool b) {
  if (a)
    c := true;
  else
    c := b;
  return c
}
```

Theorem Proving

- Step 1
  - Given the post condition, infer the weakest precondition of the program
- Step 2
  - Verify whether the given precondition can infer the weakest precondition
    - If so, the program satisfies the specification
    - Otherwise, it does not

Weakest Precondition Rules

- WP(x := E, B) = B[E/x]
- WP(s1; s2, B) = WP(s1, WP(s2, B))
- WP(if E then s1 else s2, B) = (E => WP(s1, B)) ∧ (¬E => WP(s2, B))
- WP(assert E, B) = E ∧ B
- What is the WP of our example program?
  - WP(S) = (a=>true=a∨b)∧(¬a=>b=a∨b)

- Conjecture to be proved:
  - true=> (a=>true=a∨b)∧(a=>b=a∨b)

Data Flow Analysis [5]

Peter Lee
Modified by Na Meng

Data Flow Analysis

- A technique to gather information about the possible set of values calculated at various points in a computer program

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How to do data flow analysis?

- Set up data-flow equations for each node of the control flow graph

$$out_b = trans(in_b)$$

$$in_b = join_{p \in pred_b}(out_p)$$

- Solve the equation set iteratively, until reaching a fixpoint: all in-states do not change

```
for i ← 1 to N
    initialize node i
while (sets are still changing)
    for i ← 1 to N
        recompute sets at node i
```

Work List Iterative Algorithm

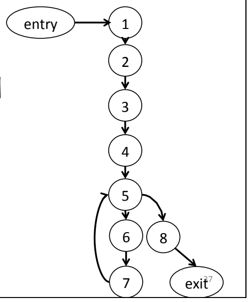
```
for i ← 1 to N
    initialize node i
    add node i to worklist
while (worklist is not empty)
    remove a node n from worklist
    calculate out-state based on in-state
    if out-state is different from the original value
        worklist = worklist U succ(n)
```

Directions of Data Flow Analysis

- Forward
  - Calculate output-states based on input-states
- Backward
  - Calculate input-states based on output-states

An Example [7]

- What variable definitions reach the current program point?



```
1: int N = input()
2: initialize array A[N + 1]
3: call check(N)
4: int I = 1
5: while (I < N) {
6:     A(I) = A(I) + I
7:     I = I + 1 }
8: print A(N)
```

Reaching Definition

- A definition at program point **d** **reaches** program point **u** if there is a control-flow path from **d** to **u** that does not contain a definition of the same variable as **d**

Reaching Definition Equations

- Forward analysis

$$out_b = gen_b \cup (in_b - kill_b)$$

$$in_b = \cup_{p \in pred_b} (out_p)$$

- gen_b: variable definitions generated by b
- $kill_b$: definitions killed at b by redefinitions of the variable(s)
- initialization: in = {}

Using Reaching Definition

- Detection of uninitialized variables

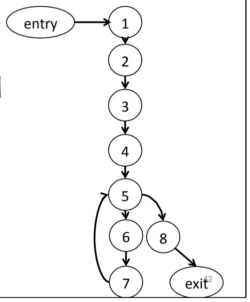
```
int x;
if (…)
   x = 1;
…
a = x;
```

Using Reaching Definition

- Loop-invariant Code Motion
  - Consider an expression inside a loop. If all reaching definitions are outside of the loop, then move the expression out of the loop

Revisit the Example[7]

- What variable definitions are or will be actually used?



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5: while (I < N) {
6:     A(I) = A(I) + I
7:     I = I + 1 }
8: print A(N)
```

Live-out Variables

- A variable **v** is **live-out** of statement **n** if **v** is used along some control path starting at **n**. Otherwise, we say that **v** is dead
  - "What variables' definitions are actually used?"

Liveness Analysis Equations

- Backward analysis

$$in_b = out_b - kill_b \cup gen_b$$

$$out_b = \cup_{p \in succ_b}(in_p)$$

- gen_b: variables used by b
- $kill_b$: if v is defined without using v, all its prior definitions are killed
- initialization: out = {}

Using Liveness Analysis

- Dead code elimination
  - Suppose we have a statement defining a variable, whose value is not used, then the definition can be removed without any side effect

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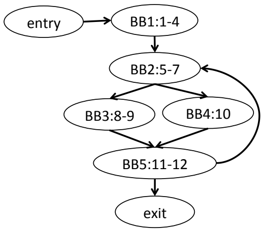
Available Expression

- An expression **e** is **available** at statement **n** if
  - it is computed along every path from entry node to **n**, and
  - no variable used in **e** gets redefined between **e**'s computation and **n**

```
1:   c = a + b
2:   d = a * c
3:   e = d * d
4:   i = 1
5:   f[i] = a + b
6:   c = c * 2
7:   if c > d goto 10
8:   g[i] = d * d
9:   goto 11
10:  g[i] = a * c
11:  i = i + 1
12:  if i <= 10 goto 5
```



- What are the available expressions at 5?
- What direction is the analysis?
- How to define gen_b and $kill_b$?
- What are the equations for in_b and out_b?

Using Available Expressions

- Common-subexpression elimination

Our analyses so far

	union	intersection
forward	Reaching definitions	Available expressions
backward	Live variables	

Questions

- Does work list iterative algorithm always terminate?

Lattices

- A lattice L is a (possibly infinite) set of values, along with ∪ and ∩ operations
  - $\forall x, y \in L, \exists$unique w and z such that $x \cup y = w$ and $x \cap y = z$
  - $\forall x, y \in L, x \cup y = y \cup x$ and $x \cap y = y \cap x$
  - $\forall x, y, z \in L, (x \cup y) \cup z = x \cup (y \cup z)$ and $(x \cap y) \cap z = x \cap (y \cap z)$
  - $\exists \bot, \mathrm{T} \in L,$ such that $\forall x \in L, x \cap \bot = \bot$ and $x \cup \mathrm{T} = \mathrm{T}$

Monotonic Functions

- The join and meet operators induce a **partial order** on the lattice elements
  - $x \subseteq y$ if and only if $x \cap y = x$
  - reflexive, anti-symmetric, transitive
- For a lattice L, a function f: L->L is **monotonic** if for all x, y $\in L$
  - $x \subseteq y \Rightarrow f(x) \subseteq f(y)$ or $x \subseteq y \Rightarrow f(x) \supseteq f(y)$

Reaching definition is monotonic

$$out_b = gen_b \bigcup (in_b - kill_b)$$

- Proof (for single-variable single-block programs) by contradiction:
  - Suppose $\mathrm{in}_b = \{1\}, out_b = \{0\}$, where 1 means there is a variable definition, 0 means no definition, then $gen_b = \{0\}, kill_b = \{1\}$.
  - However, $kill_b = \{1\}$ only if the block **b** has a redefinition of the variable, which means $gen_b = \{1\}$

- Therefore, after limited number of iterations (N* (E+1) at worst case), every definition is propagated to every node
- Therefore, we can find a fixpoint p, such that f(p) = p

In dataflow analysis, we require that all flow functions be monotone and have only finite-length effective chains

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Ingredients of a Dataflow Analysis

- Flow direction
- Transfer function
- Meet operator (Join function)
- Dataflow information
  - Set of definitions, variables, and expressions
  - Initialization
  - How about concrete data values?

Inter-procedural Analysis [8]

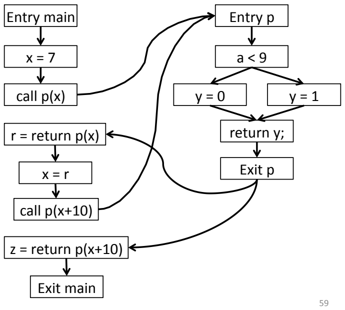
Stephen Chong
Imported by Na Meng

Procedures

- So far we have looked at **intra-procedural** analysis: analyzing a single procedure
- **Inter-procedural** analysis uses calling relationships among procedures
  - Connect intra-procedural analysis results via call edges
  - Enable more precise analysis information

Inter-procedural CFG

```
void main() {
    x = 7;
    r = p(x);
    x = r;
    z = p(x+10);
}
int p(int a) {
    int y;
    if (a < 9)
        y = 0;
    else
        y = 1;
    return y;
}
```

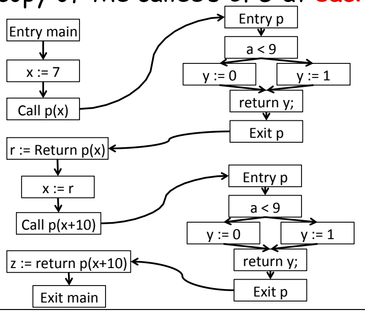


Imprecision

- Dataflow facts from one call site can "taint" results at other call sites
  - Is **z** a constant?

Inlining

- Make a copy of the callee's CFG at **each** call site



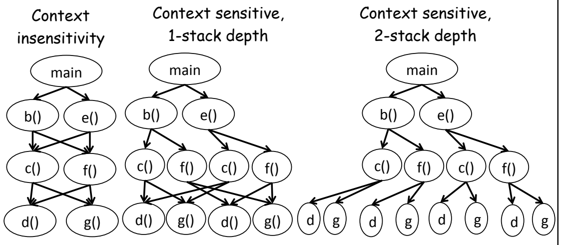
Exponential Size Increase

- How about recursive function calls?
  - p(int n) {… p(n - 1);…}
- The exponential increase makes analysis infeasible

Context Sensitivity

- Make a **finite** number of copies
- Use context information to determine when to share a copy
  - Different decisions achieve different tradeoffs between precision and scalability
- Common choice: approximation call stack

An Example



Procedure Summaries

- In practice, people don't construct a single global CFG and then perform dataflow
- Instead, construct and use **procedure summaries**
- Summarize effect of callees on callers
  - E.g., is there any side effect on callers?
- Summarize effect of callers on callees
  - E.g., is any parameter constant?

Other Contexts

- Object/pointer sensitivity
  - What is the type of a given object and what are the corresponding possible method targets?
  - What is the value of a given object's field?

Pointer Analysis

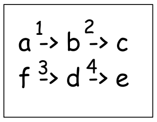
- What is the points-to set of p?
  int x = 3;
  int y = 0;
  int* p = unknown() ? &x : & y;
- Alias analysis
  - Decide whether separate memory references point to the same area of memory
  - Can be used interchangeably with pointer analysis (points-to analysis)

Flow Sensitivity

- Flow insensitive analysis
  - Perform analysis without caring about the statement execution order
    - E.g., analysis of c1;c2 will be the same as c2;c1
    - Address-taken, Steensgaard, Anderson
- Flow sensitive analysis
  - Observes the statement execution order

An Example

1: a = &b
2: b = &c
3: f = &d
4: d = &e
5: a = f

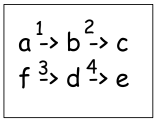


- After 5, both ***a** and ***f** point to **d**

Address Taken

- Assume that variables whose addresses are taken **may** be referenced by all pointers
  - Address-taken variables: b, c, d, e
  - A single alias pointer set: {a, b, f, d}

1: a = &b
2: b = &c
3: f = &d
4: d = &e
5: a = f

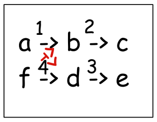


Steensgaard

- Constraints
  - p = &x: x$\in$pts-to(p)
  - p = q: pts-to(p) = pts-to(q)
  - p = *q $\forall a \in$pts-to(q), pts-to(p)=pts-to(a)
  - *p = q $\forall b \in$pts-to(p), pts-to(b)=pts-to(q)

1: a = &b
2: b = &c
3: f = &d
4: d = &e
5: a = f

  - Points-to set: pts(a) = pts(f) ={b, d}

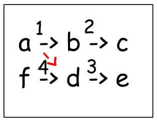


Andersen

- Subset Constraints
  - p = &x: x$\in$pts-to(p)
  - p = q: pts-to(q)$\subseteq$pts-to(p)
  - p = *q $\forall a \in$pts-to(q), pts-to(a)$\subseteq$pts-to(p)
  - *p = q $\forall b \in$pts-to(p), pts-to(q)$\subseteq$pts-to(b)

1: a = &b
2: b = &c
3: f = &d
4: d = &e
5: a = f

  - Points-to set: pts(a)={b, d}, pts(f)={d}



Flow-sensitive Pointer Analysis

$$out_b = gen_b \cup (in_b - kill_b)$$
$$in_b = \cup_{p \in pred_b} (out_p)$$

- x = y: **strong update**
  - kill—clear pts(x)
  - gen—add pts(y) to pts(x)
- *x = y:
  - If x definitely points to a single concrete memory location z, pts(z) = y (strong update)
  - If x may point to multiple locations, then **weak update** by adding y to pts of all locations

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