# Some NP-Complete Problems

**SATISFIABILITY(SAT)**
INSTANCE: A Boolean expression $E$ over variables $x_1, x_2, \ldots, x_n$ in conjunctive normal form.
QUESTION: Is there an assignment of truth values to $x_1, x_2, \ldots, x_n$ making $E$ true?

**3-SAT**
INSTANCE: A Boolean expression $E$ in conjunctive normal form such that each clause contains exactly 3 literals.
QUESTION: Is there a satisfying assignment for $E$?

**3-COLORABILITY**
INSTANCE: Graph $G = (V, E)$.
QUESTION: Is $G$ 3-colorable, that is, is there a function $f : V \rightarrow \{\text{red}, \text{blue}, \text{green}\}$ such that $f(u) \neq f(v)$ whenever $(u, v) \in E$?

**3-DIMENSIONAL MATCHING (3DM)**
INSTANCE: A set $M \subset W \times X \times Y$ where $W$, $X$, and $Y$ are disjoint sets having the same number $q$ of elements.
QUESTION: Does $M$ contain a matching, i.e., a subset $M' \subset M$ such that $|M'| = q$ and no two elements of $M'$ agree in any coordinate?

**EXACT COVER BY 3-SETS (X3C)**
INSTANCE: Finite set $X$ with $|X| = 3q$, $q$ an integer; collection $C$ of 3-element subset of $X$.
QUESTION: Does $C$ contain an exact cover for $X$, i.e., a subcollection $C' \subset C$ such that every element of $X$ occurs in exactly one member of $C'$?

**PARTITION**
INSTANCE: A finite set $A$, and a "size" $s(a) \geq 0$ defined for each $a \in A$.
QUESTION: Is there a subset $A' \subset A$ such that

$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)?$$

**KNAPSACK**
INSTANCE: Items $1, \ldots, N$ with $size(i) \geq 0$ and $value(i) \geq 0$ defined for each item $i$; integers $M, K \geq 0$.
QUESTION: Is there a subset $S \subset \{1, \ldots, N\}$ such that

$$\sum_{i \in S} size(i) \leq M$$

and

$$\sum_{i \in S} value(i) \geq K?$$

**CLIQUE**
INSTANCE: Undirected graph $G = (V, E)$, positive integer $K \leq |V|$.
QUESTION: Does $G$ have a clique of size $K$ or more, i.e., a subset $V' \subset V$ with $|V'| \geq K$ such that every two vertices of $V'$ are adjacent?

**INDEPENDENT SET**
INSTANCE: Undirected graph $G = (V, E)$; positive integer $K \leq |V|$.
QUESTION: Does $G$ contain an independent set of size $K$ or more, i.e., a subset $V' \subset V$ such that $|V'| \geq K$ and such that no two vertices of $V'$ are adjacent?

**VERTEX COVER (VC)**
INSTANCE: Undirected graph $G = (V, E)$; positive integer $K \leq |V|$.
QUESTION: Is there a vertex cover of size $K$ or less for $G$, i.e., a subset $V' \subset V$ such that $|V'| \leq K$ and such that for each $(u, v) \in E$, either $u \in V'$ or $v \in V'$?

**DOMINATING SET**
INSTANCE: Undirected graph $G = (V, E)$; positive integer $K \leq |V|$.
QUESTION: Does $G$ contain a dominating set of size $K$ or less, i.e., a subset $V' \subset V$ with $|V'| \leq K$ such that for all $u \in V - V'$ there is a $v \in V'$ for which $(u, v) \in E$?

**HAMILTONIAN CIRCUIT (HC)**
INSTANCE: Undirected graph $G = (V, E)$.
QUESTION: Does $G$ contain a Hamiltonian circuit, i.e., a simple cycle of length $|V|$?

End of List